Project 1: Application of s = rθ and v = rω in Two Pulley Systems

Pulley systems provide an ideal environment for using the formulas s = rθ and v = rω . Most of these problems can be solved more quickly by using ratios (which can be justified by the s = rθ and v = rω formulas) but ratios obscure the insight to the meaning of the relations between r, θ and ω gained by using the formulas.

The essential concept in our work with pulley systems is:

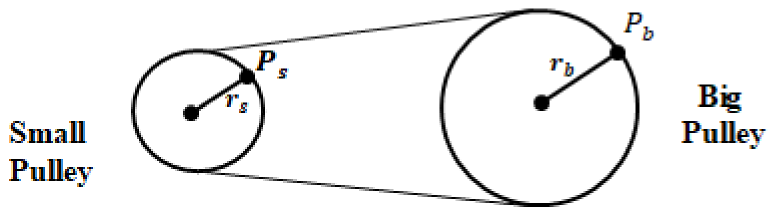

Fig. 1

UNDERLYING PRINCIPLE: If the connecting belt moves without slipping (we will always assume this is the case), a point on the rim of the small wheel, $\boldsymbol{P_s}$, must have the same velocity and move the same distance as a point on the rim of the large wheel, $\boldsymbol{P_b}$. We will illustrate how this principle allows us to solve pulley problems in the following illustrative example.

Sample Example: All of the parts of this example will deal with a pulley system consisting of a 2" wheel and a 7" wheel as shown below.

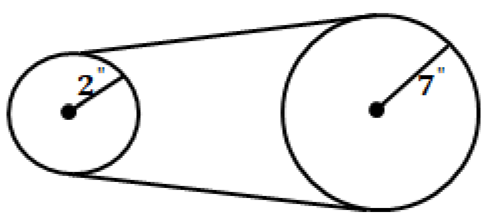

Fig. 2

a) If the small wheel turns through an angle of 50°, what angle (expressed in degrees) does the large wheel turn through?
b) If the small wheel spins at the rate of 6 rev. per minute, how many rpms is the large wheel making?

Solution to part a):
UNDERLYING PRINCIPLE: The distance a point on the rim of a small wheel moves is the distance a point on the rim of the large wheel moves.

Since the pulley example in part a) doesn't involve time or speed, but just distances and angles, the underlying principle gives the link" between the small and large wheels, namely that the point on the rim of either wheel moves the same distance.

Step (1): Converting θ from degrees to radians
We first compute the distance moved by a point on the rim of the small wheel for which we use:

$$\overbrace{s}^{\text{distance}} = r\,\theta$$

where θ must be measured in radians and the distance units for s and r must agree.

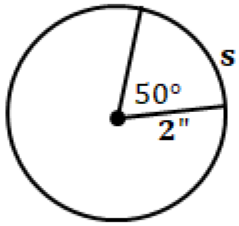

Fig. 3

Note that $\theta = 50°$ must be changed to radians before using the formula $s = r\theta$, so

$$\theta = (50\,\cancel{deg})\left(\frac{\pi\,rad}{180\,\cancel{deg}}\right)$$
$$= \frac{50\pi}{180}\,rad$$
$$= \frac{5\pi}{18}\,rad$$

Step(2): Solving for the distance the small wheel moves.

Thus substituting $\frac{5\pi}{18}$ for θ and 2 for r we get

$$s = r\theta$$
$$s = (2)\left(\frac{5\pi}{18}\right)$$
$$s = \frac{5\pi}{9}\,inches$$

Recall: in the formula $s = r\,\theta$, the distance unit are the same for r and s. θ must be in radians

Step (3): Finding the distance the large wheel moves.

Now referring to the diagram, below we have completed steps (1) and (2) and are about to move our computations to the large wheel using step (3) and that the <u>underlying principle</u> that the distances moved by points on the rim of either wheel are equal.

$$\boldsymbol{\theta\ in\ deg \rightarrow \theta\ in\ rad \rightarrow s_{small\ wheel} = s_{big\ wheel} \rightarrow \theta\ in\ rad \rightarrow \theta\ in\ deg}$$

(1) (2) (3) (4) (5)

Therefore, we can state that: $\mathbf{s_b = s_s = \frac{5\pi}{9}}$ **inches.**

Step (4): ***Solving for θ on the large wheel:***

Since from the underlying principle a point on the rim of the large wheel must <u>also</u> move $\frac{5\pi}{9}$ *inches*, we have for the large wheel:

$$s = r\theta$$
$$\frac{5\pi}{9} = 7\theta$$
$$\theta = \frac{5\pi}{63}\,radians$$

Note: s and r both are expressed in inches in the formula $s = r\,\theta$. Therefore θ will result in the units radians.

Step (5): ***To change θ from radians to degrees***

$$\theta = \left(\frac{5\pi}{63}\,rad\right)\left(\frac{180\,deg}{\pi\,rad}\right)$$
$$= \frac{5}{63}(180)\,deg$$
$$= \frac{100}{7}\,deg$$

Solution to part b):

UNDERLYING PRINCIPLE: The velocity of a point on the rim of the large wheel must equal the velocity of a point on the rim of the small wheel. Starting with the fact that the small wheel is making 6 rpm, we will compute the velocity of a point on its rim using $\mathbf{v} = \mathbf{r\omega}$. Then the diagram which follows will lead to the large wheel's angular velocity ω. We use the underlying principle that since the example deals with the speed (rpms), the "link" between the small wheel and the large wheel is that the velocity of a point on the rim of either wheel is the same.

Thus to move our computations from the small wheel to the large wheel, we must compute the velocity of a point on the rim of the small wheel. To do so, we use v = rω which requires that we know ω, the wheel's angular velocity.

Step (1): getting ω for the small wheel:

If the small wheel is making 6 rpm, then:

$$\omega_s = 6\frac{rev}{min}\left(\frac{2\pi\, rad}{1\, rev}\right) \Leftrightarrow 12\pi\frac{rad}{min}$$

Note: ***we convert from*** $\frac{\boldsymbol{rev}}{\boldsymbol{min}}$ ***by multiplying by the "units conversion fraction"*** $\frac{\boldsymbol{2\pi\, rad}}{\boldsymbol{1\, rev}}$.

Thus $\omega_s = 12\pi\ \frac{rad}{min}$

Step (2): solving for the linear velocity $\mathbf{v}_s$ for the small wheel:

$$\begin{aligned} v_s &= r_s\omega_s \\ &= (2)(12\pi) \\ &= 24\pi\frac{in}{min} \end{aligned}$$

Note: ***v_s is in inches per minute because from the formula: $v = r\omega$, we see that the units for v and r must agree with those of r and be in inches and its time unit must agree with the time units for ω which is in*** $\frac{\boldsymbol{rad}}{\boldsymbol{min}}$.

Thus a point on the rim of the small wheel has a velocity of $\mathbf{v} = \boldsymbol{24\pi}\frac{\boldsymbol{in}}{\boldsymbol{min}}$.

In the diagram below, the comment under the step numbers indicate how that step is accomplished.

$$\boldsymbol{rpm_{small\ wheel} \xrightarrow[\text{change of units fractions}]{(1)} \omega_{small\ wheel} \xrightarrow[v = r\omega]{(2)} v_{small\ wheel} \underset{\text{underlying principle}}{\overset{(3)}{=}} v_{large\ wheel} \xrightarrow[v = r\omega]{(4)} \omega_{large\ wheel} \xrightarrow[\text{change of units fractions}]{(5)} rpm_{large\ wheel}}$$

Step (3): At this point, we invoke the underlying principle which states that the speed of a point on the rim of one wheel is the same as the speed of a point on the other wheel's rim.

Accordingly, we now know that the speed of a point on the rim of the large wheel is also $24\pi\frac{in}{min}$.

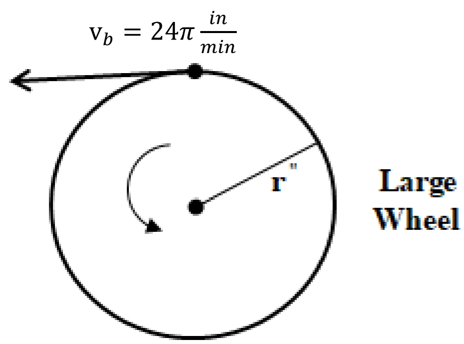

Fig. 4

Step (4): getting ω for the large wheel.

$$v_b = r_b \omega_b$$
$$24\pi = 7\omega_b$$
$$\omega_b = \frac{24\pi}{7}\frac{rad}{min}$$

Note: **ω_b is in radians per minute because from the formula: $v = r\omega$, we see that the *units for v and r must agree in units of length which are in inches and v and r must agree in units of time, therefore if v is in minutes, then ω will be in units of minutes as well. Also, the default units for ω is $\frac{rad}{time}$.***

Step (5): ***To change ω_b from $\frac{rad}{min}$ to rpm.***

$$\boldsymbol{\omega_b = \frac{24\pi}{7}\frac{rad}{min}\left(\frac{1\,rev}{2\pi\,rad}\right) \Leftrightarrow \frac{24\pi}{14\pi}\frac{rev}{min} = \frac{12}{7}\frac{rev}{min}}$$

So the large wheel makes $\frac{12}{7}$ rpm.

Classwork Exercises:

1. In the pulley system below, if the big wheel turns through an angle of 20°, through what angle must the small wheel turn? Write your answer in terms of degrees.

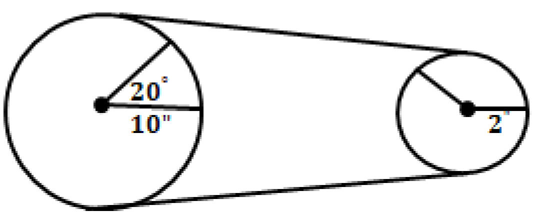

Fig. 5

2. In the diagram below, if the big wheel makes $2\frac{1}{2}$ revolutions, how many revolutions does the small wheel make?

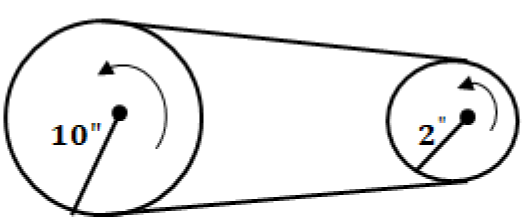

Fig. 6

Homework Exercises:

Note: DO NOT USE RATIOS. Work each problem by finding, as an intermediate step, the distance moved by points on the rim of the wheel or the speed of points on the rim of the wheel.

1. In a 8"and 3" pulley system, the large wheel turns through 2 rpm. Find the rpm of the smaller wheel.

2. In a 8" and 5" pulley system, the 5" wheel turns through an angle of 30°. Find the angle (in degrees) which the 8" wheel turns through.

3. A pulley system consists of a 10 inch wheel and another whose radius is unknown. A point on the rim of the wheel of unknown radius moves 450 inches when it revolves through an angle of 225°. Find the angle (in degrees) the 10 inch wheel turns through.

4. A pulley system consists of one wheel of radius 4" and another whose radius is unknown. If the 4" wheel turns through an angle of 20° and the other wheel turns through an angle of 60°, find the radius of the unknown wheel. (Be sure to perform the intermediate step requested in the instructions.)

5. In a 5" and 7" pulley system, the velocity at a point on the rim of the 7" wheel is $10\,\frac{\text{in}}{\text{min}}$. Find the *rpms* being made by the 5" wheel.

6. A point on the rim of a wheel of unknown radius in a pulley system has a velocity of $16\,\frac{in}{min}$. The wheel is making 4 rpms. If the radius of the other wheel is 8 *inches*, find the 8" wheel's rpms and the unknown wheel's radius.

7. The big wheel of a pulley system turns at 2 rpm and the small wheel turns at 5 rpm. If a point on the rim of the large wheel moves with a speed of $10\,\frac{in}{min}$, find the radius of each wheel.