CALCULUS 211-FINAL EXAM-DECEMBER 15, 2004

1) Determine the equation of the plane formed by the intersecting lines
\[ \frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{4} = t \text{ and } \frac{1-x}{1} = \frac{2-y}{2} = \frac{z+1}{2} = t \]

2) Evaluate \( \frac{\partial z}{\partial x} \) at the point (2,2,1) for the surface \( x\ln z + xyz^2 + zy = 5 \)

3) Determine, for \( w = x^2y^3 \sin z + x \) at the point (2, 1, \( \frac{\pi}{6} \))
   a) Maximum value of the directional derivative \( \frac{dw}{ds} \)
   b) The equation of the plane tangent to the surface \( x^2y^3 \sin z + x = 4 \) at the same point

4) For a point moving along the space curve \( x = t^2 + 1 \), \( y = \cos t \), \( z = e^{2t} \)
   determine the cosine of the angle between the position and acceleration vectors, at t=0

5) Find the critical points and classify them for \( z = x^4 - 8x^2 + y^2 - 4y \)

6) Using Lagrange multipliers, find the point on the line \( y = -2x + 4 \) that is closest to point (0,1)
   (hint: minimize the square of the distance between the points)

7) Evaluate the double integral \( \int_0^1 \int_0^1 \left( \frac{1}{\sqrt{1+4y^3}} \right) dy \) by reversing the order of integration

8) Evaluate the volume in the region bounded by the parabolic cylinder \( y = x^2 \) and the planes
   \( y + z = 1 \) and \( z = 0 \), by evaluating the volume integral \( \iiint dV \)

9a) Determine the potential of the conservative vector field \( \mathbf{F} = xz^2 \mathbf{i} + 2yz \mathbf{j} + x^2z \mathbf{k} \)
   b) Evaluate the work done by this vector field in moving along an object from
      the point (0,0,1) to (1,2,1)

10) For the space curve \( x = t^2 + 1 \), \( y = \frac{t^4}{4} \), \( z = t^3 - 1 \)
    evaluate the line integral \( \int_c \mathbf{F} \cdot d\mathbf{R} \) for \( \mathbf{F} = 5y \mathbf{i} + 7z \mathbf{j} + x \mathbf{k} \) between \( 0 \leq t \leq 1 \)

11) For the vector field \( \mathbf{F} = -yi + xyj \) evaluate as a line integral \( \oint \mathbf{F} \cdot d\mathbf{R} = \oint -y dx + xy dy \)
    around the region enclosed by the curves \( y = 4x \) and \( y = x^3 \) in the first octant.

12) Using Greens Theorem evaluate the integral \( \oint \mathbf{F} \cdot d\mathbf{R} \) in problem (11)
    by double integration.