PHD DISSERTATION DEFENSE ANNOUNCEMENT

AN EFFICIENT BOUNDARY INTEGRAL METHOD FOR STIFF FLUID INTERFACE PROBLEMS

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ABSTRACT

The purpose of this thesis is to formulate and investigate a boundary integral method for the solution of the internal waves/Rayleigh-Taylor problem. This problem describes the evolution of the interface between two immiscible, inviscid, incompressible, irrotational fluids of different density in three dimensions. A mathematical model of this interfacial flow problem in 3D is derived. The motion of the interface and fluids is driven by the action of a gravity force, surface tension at the interface, elastic bending and/or a prescribed far-field pressure gradient. The presented models include derived equations for the evolution of the interface and dipole density on the interface. The interface is a generalized vortex sheet, and dipole density is interpreted as the (unnormalized) vortex sheet strength. Presence of the surface tension on elastic bending effects introduces high order derivatives into the evolution equations. This makes the considered problem stiff and the application of the standard explicit time-integration methods suffers strong time-step stability constraints. The derived initial value problem for the Rayleigh-Taylor and hydroelastic waves problem in three dimensions is solved using the proposed efficient numerical method.

The proposed numerical method employs a special interface parameterization that enables the use of an efficient implicit time-integration method via a small-scale decomposition. This approach allows one to capture the nonlinear growth of normal modes for the case of Rayleigh-Taylor instability with the heavier fluid on top. Linear stability analysis is performed and the numerical results for the nonlinear problem are validated by comparison to the obtained analytic solution of the linearized problem for a short time. Further validation is done by checking the energy and the interface mean height preservation. The developed model and numerical method can be efficiently applied to study the motion of internal waves for doubly periodic interfacial flows with surface tension and elastic bending stress at the interface.

The thesis has three parts. The nondimentsonalized governing equations for the interfacial fluid flow with surface tension and elasticity are presented in the part 1 of the thesis. This part also includes the equations for the elastic interface position, interface density and the equations for the fluids normal and tangential velocities. In part 2 the small-scale decomposition for the normal velocity integral is performed. A leading order approximation of the singular velocity integral, known as a Birkhoff-Rott integral, is derived. The leading order part consists of the terms in governing equations that are dominant at high wavenumber and are responsible for the numerical stiffness (equivalently these are the high derivative terms). These terms are contained in a nonlocal operator. The leading order approximation is expressed in terms of the Riesz transforms, and it is computed with spectral accuracy using the FFT method. Then the expressions for the mean height of the interface and the total energy of the system are derived. These quantities are the invariants of the interface motion. The symmetry property of the problem solution is studied. Finally, in part 3 the semi-implicit numerical method is presented. It includes the discussion of the integration method for the interface velocity, including the fast Ewald summation algorithm for the velocity integral, and includes the results of numerical simulations for the fluids interface evolution.