CONTRIBUTED PAPERS

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The Coupled Motion of Flexible Bodies and Vortex Sheets in Inviscid Flows

This poster will present a classification of the typical motions of 1-D flexible bodies ("beams") undergoing arbitrary motions in high-Reynolds-number fluid flows. Previous works have generally focused on rigid bodies and small-amplitude motions. We have developed an implicit numerical scheme coupling the motion of a flexible fiber to ambient vorticity shed from the fiber edges and advected with the fluid flow. The scheme is computationally inexpensive and can compute flows past a variety of active and passive flexible bodies to model flapping flags, swimming fish, falling sheets, and myriad other phenomena of recent interest to theoretical mechanicians.

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Heat Transfer Problem in Cryosurgery

A heat transfer problem involving contact freezing of biological tissue in cryosurgery is investigated. Although the precise nature of the mechanism, biochemical and/or mechanical, responsible for the destruction of cells in contact freezing is not completely understood, it is known that the formation of a cryolesion occurs within the boundaries of the freeze-zone, and that the factors affecting cell injury during any freezing/thawing cycle include the coldest temperature in the tissue, the duration of the freezing/thawing cycle, the rate at which the freezing front moves through the tissue, and the thawing rate.

The problem is modeled by a moving boundary problem of the Stefan-type with body heating terms and phase-change which does not occur at a fixed temperature, but over a temperature range. The effects of metabolic heat generation and blood perfusion in the tissue are taken into account in the model. An implicit finite-difference scheme, based on the weak solution (enthalpy) method, is used in the model to determine the temperature distribution in the tissue surrounding the cryogenic probe and the location of the freezing front boundary with respect to time.

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Electric Field Induced Deformation and Breakup of a Drop in a Microfluidic Device

Fluid can be transported in microfluidic devices in an either continuous or digital fashion. The latter uses droplets which can be moved, deformed, split or fused for applications ranging from biochemical assays to drug delivery. The key is often to predict and control droplet motion and deformation in a precise manner. We present results on the detailed deformation of a liquid drop immersed in a surrounding fluid and subjected to the application of a uniform electric field, and identify features particular to the confinement of the surrounding fluid in a small chamber. For this purpose, we use direct numerical simulations taking into account both hydrodynamic and electrostatic forces, in which the droplet and its surrounding fluid are moved and deformed using the fundamental equations of motion. The interface is tracked by the level set method and the electrostatic forces are computed using the Maxwell stress tensor. Our results show how the drop deforms under the action of non-uniform stresses acting on its surface before eventually breaking into two parts, and how such a deformation and breakup are influenced by the proximity of the device walls. A good agreement with previous analytical results is found for small drop deformations, a small dielectric mismatch between the drop and the ambient fluid and an infinite computational domain. When these conditions are relaxed, however, the discrepancy can be significant. Joint work with Pushpendra Singh.

Gerard Awanou and Ming-Jun Lai

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Trivariate Spline Approximations of 3D Navier-Stokes Equations

We present numerical approximations of the 3D steady state Navier-Stokes equations in velocity-pressure formulation using trivariate splines of arbitrary degree d and arbitrary smoothness $r < d$. Using functional arguments, we derive the discrete Navier-Stokes equations in terms of B-coefficients of trivariate splines over a tetrahedral partition of any given polygonal domain. Smoothness conditions, boundary conditions and the divergence-free condition are enforced through Lagrange multipliers. The pressure is computed by solving a Poisson equation with Neumann boundary conditions. We have implemented this approach in MATLAB and present numerical evidence of the convergence rate as well as experiments on the lid-driven cavity flow problem.

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Prediction of Polyadenylation Sites Using Support Vector Machines

Messenger RNA polyadenylation is an essential cellular process for the maturation of most mRNAs in eukaryotes. A large number of human and mouse genes have multiple polyadenylation sites (referred to as poly(A) sites) that lead to variable gene transcripts with distinct functions. It is generally accepted that polyadenylation is determined by the sequence surrounding the poly(A) site. Various methods have been developed to predict poly(A) sites with moderate success. Using 15 cis-regulatory elements surrounding the poly(A) site and the support vector machine (SVM), we have developed a highly accurate method to predict poly(A) sites. This tool can greatly facilitate research on eukaryotic genes and genomes. Joint work with Robert M. Miura.

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Asymptotic Structure of Strongly Nonlinear, Laminar Langmuir Circulation

Coherent structures are responsible for much of the turbulent transport of heat, mass and momentum in configurations ranging from Rayleigh--Benard convection to wall-bounded shear flows. Thus, there is significant interest in predicting the form and dynamics of these flow structures from the governing conservation equations. Waleffe (2001) has coined the phrase ``exact coherent structures'' to describe laminar but fully nonlinear, exact solutions to the Navier--Stokes equations that exhibit striking similarities with coherent structures observed in wall-bounded turbulent shear flows, including cross-flow-periodic vorticity aligned with the mean shear. In other flows including Rayleigh--Benard convection and its cousins, remnants of a primary instability mode are clearly evident in the turbulent state. Generally, numerical (e.g. continuation and homotopy) methods must be employed to extract the modal structure at large Reynolds or Rayleigh numbers. For example, the majority of analytical studies of non-rotating Rayleigh--Benard convection and Langmuir circulation (LC) -- a wind and wave-driven convective flow commonly observed in natural water bodies -- have been restricted to the linear or weakly nonlinear regime. Here, matched asymptotic analysis and global conservation constraints are used to obtain a semi-analytic description of strongly nonlinear LC in the small ``Langmuir number'' (La) limit (where La is an inverse Reynolds number); a simple analytical expression is obtained for the nonlinear dispersion relation between the LC vortex amplitude and horizontal wavenumber. Although the asymptotic solutions are likely to be unstable, they exhibit flow features found in turbulent LC, including the complete vertical re-distribution of the downwind momentum contained in the base Couette flow -- in marked contrast to weakly nonlinear convection rolls.

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Time Stability of Solitary-Wave-Like Waves of a Two Layer Fluid over a Bump

We consider waves at the interface of a two-layer incompressible fluid in a two dimensional domains with rigid boundaries. By assuming that lower boundary contains small obstructions, A Forced Modified KdV equation is derived when the KdV Theory fails and four types of symmetiric time independent solitary-wave-like solutions are found numerically according to the perturbation of horizontal velosity at far ustream. Time stability of such symmetric solitary-wave-like solutions is studied.

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A Numerical Study of the Evolution of Nonlinear Water Waves

We study the evolution of nonlinear surface gravity-capillary waves interacting with slowly-varying surface currents using a system of coupled nonlinear evolution equations. The system is solved numerically using a pseudo-spectral method and its numerical solutions are compared with solutions of the Euler equations and other model equations. For oceanic applications, the evolution of irregular waves are also considered and the model is generalized to include slowly-varying bottom topography effects. Joint work with David Lyzenga (Univ. of Michigan), Christopher Kent (Florida Inst. of Tech.), and Yuan-Nan Young (NJIT).

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Lifted Laminar Aerosol Flames: Experiments and Interpretations

Combustion in multiphase systems is ubiquitous in practical applications, but understanding of droplet and particle combustion that involves interaction of multiple flames is lacking. Models under development require input and validation from carefully designed measurements, but respective experiments with spray or cloud combustion are difficult to process quantitatively. Measurements of such key parameters as individual particle or droplet ignition delay, burn time, flame size and temperature are needed, but usually not available. A novel experimental approach discussed in this poster uses lifted laminar flame burners that enable one to establish and study stable flames of solid powders, liquid fuels, or even slurries. Experimental approach and initial results for several multiphase flame systems will be presented and discussed.

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Combining Synaptic and Cellular Resonance in a Feed-Forward Neuronal Network

We derive a mathematical theory to explain the subthreshold resonance response of a neuron to synaptic input. The theory shows how a neuron combines information from its intrinsic resonant properties with those of the synapse to determine the neuron's generalized resonance properties. Our results show that the maximal response of a postsynaptic neuron can lie between the preferred intrinsic frequency of the neuron and the synaptic resonance frequency. We compare our theoretical results to parallel findings on experiments of the crab stomatogastric ganglion. Joint work with Vahid Tohidi (Rutgers-Newark) and Amitabha Bose and Farzan Nadim (NJIT).

Leo Espin

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Flow in Pulsating Channels in the Presence of a Horizontal Pressure Gradient

The purpose of this work is to study a model for the two-dimensional flow between parallel walls produced by oscillatory displacement of one of the walls. In addition, a pressure gradient is present providing a net horizontal flow also. We consider solutions of the Navier-Stokes equations for this configuration which are exact in the sense that they can be described by solutions of one-dimensional partial differential equations. The case of the flow for a small Reynolds number is covered analytically and the numerical problem for order one Reynolds numbers is posed. Numerical results are presented for a range of Reynolds numbers.

The exact solution is a consequence of a stagnation-type flow and a decoupling between the vertical and horizontal components of velocity is possible. It has been shown in previous work that the oscillatory flow can become chaotic, for example, and in such situations the throughflow due to the pressure gradient is forced by aperiodic coefficients. In the special case of time-periodic solutions, the horizontal velocity is governed by a partial differential equation with periodic coefficients.

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On Traveling Waves in Porous Media Combustion with Small Thermal Diffusivity

Sivashinsky's model of subsonic detonation describes the propagation of combustion fronts in highly resistable media. The combustion front is described by a traveling wave solution of the corresponding system of equations. It is known that there exists a traveling wave asymptotically connecting the unburnt and burnt states, which is unique if thermal diffusivity is neglected as shown by Gordon, Kamin and Sivashinsky. The question of whether the wave is unique in the presence of thermal diffusivity has remained open. We resolve this issue through applying geometric singular perturbation theory. Joint work with Christopher K.R.T Jones (University of North Carolina at Chapel Hill & SAMSI) and Peter Gordon (Department of Mathematical Sciences, New Jersey Institute of Technology).

Roy Goodman

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The n-bounce Resonance in Wave Interactions

The *n*-bounce resonance is a phenomenon that has been seen in the interaction of waves with localized structures and between pairs of nonlinear waves. A slowly moving wave incident upon a defect may be captured into a standing wave, while waves with certain “resonant” initial velocities are reflected or transmitted after interacting *n* times (a.k.a. "bouncing"). These behaviors are intertwined in a complicated pattern that appears fractal. A new framework, involving Melnikov integrals and matched asymptotic expansions, is developed for the study of ordinary differential equations describing this phenomenon, yielding new quantitative results for a few such systems, including critical velocities for capture and an explanation, via iterated maps, for the fractals that appear.

Roy Goodman, Daniel Goldman, Lou Kondic, Bruce Bukiet, Michael Booty, and Michael Siegel

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Modules for an Undergraduate Capstone Course

We give a brief overview of the ideas and principles that underlie the senior-level undergraduate Capstone course in Applied Mathematics at NJIT, and describe some of the various modules combining theory and experiment that have been used over the last few years with others that are planned for implementation in the near future. Past and current funding for lab equipment has been provided by the National Science Foundation, and many members of the Department of Mathematical Sciences have been active in the evolution of the course and lab.

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A Stretch-Temperature Model for Flame-Flow Interaction

The classical relation between the flame speed and the stretch, employed in modeling flame-flow interaction, is valid only for positive Markstein lengths (high Lewis numbers). At negative Markstein lengths (low Lewis numbers) the corresponding dynamical system suffers short-wavelength instability, making the associated initial value problem ill-posed. In this study the difficulty is resolved by incorporation of higher-order effects. As a result one ends up with a reduced model based on a coupled system of second-order dynamic equations for the flame interface and its temperature. As an illustration the new model is applied for description of diffusively unstable stagnation-point flow flames. This is a joint work with G. Sivashinsky and M. Frankel.

Arnaud Goullet[1], Rodolphe Chabreyrie[1], Nadine Aubry[1], Tounsia Benzekri, Cristel Chandre, Ricardo Lima, and Michel Vittot

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Control of Chaotic Advection via a Hamiltonian Approach

Chaotic advection is often used as a stirring means in laminar flows where turbulent mixing is absent. While numerous works have focused on enhancing chaos to increase mixing, many applications require

both enhancement and suppression in order to fully control reaction rates. In this work, we are interested in controlling chaotic advection in the case of a two-dimensional incompressible fluid flow. The latter can then be described by a Hamiltonian function which coincides with the streamfunction. Our control technique is inspired by that recently developed for Hamiltonian systems, which consists in introducing a suitable small term in the original Hamiltonian function. Such a control induces a small secondary flow that hardly alters the structure of the flow itself in terms of its streamlines, but is capable of drastically changing the dynamical behavior of passive particles. By carefully choosing the control term, we show the possibility of either reducing the chaoticity, by creating regular islands and barriers, or increasing it within a confined region in the phase space. We demonstrate here the efficiency of the technique on two different flow configurations, Rossby Waves and Rayleigh-Bénard Convection, which can both exhibit chaotic advection under certain conditions. In the two cases, we derive a suitable control term and show its effect on chaotic advection through numerical simulations of the material lines and Poincaré sections. We also show the robustness of the control with respect to external white noise. Joint work with Rodolphe Chabreyrie and Nadine Aubry (NJIT); Tounsia Benzekri, Cristel Chandre, Ricardo Lima, and Michel Vittot (Centre de Physique Théorique, France).

David J. Horntrop

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Spectral Schemes for Stochastic Partial Differential Equations

The use of stochastic partial differential equations as mathematical models is becoming increasingly widespread in many areas of application, including mathematical fluid dynamics. In this poster, spectral schemes for the numerical simulation of solutions to stochastic PDE's will be described. This algorithmic approach is verified through computational studies of some exactly solvable benchmark problems involving both additive and multiplicative noise.

M. Janjua, S. Nudurupati, P. Singh, and N. Aubry

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Direct Numerical Simulation (DNS) of Suspensions in Spatially Varying Electric Fields

We have developed a numerical scheme to simulate the motion of dielectric particles suspended in a dielectric liquid in uniform and nonuniform electric fields. The particles are moved using a direct simulation scheme in which the fundamental equations of motion of fluid and solid particles are solved without the use of models. The motion of particles is tracked using a distributed Lagrange multiplier method (DLM). One of the key features of the DLM method is that the fluid-particle system is treated implicitly by using a combined weak formulation where the forces and moments between the particles and fluid cancel, as they are internal to the combined system. The flow inside the particles is forced to be a rigid body motion using the distributed Lagrange multiplier method. In our numerical scheme the Marchuk-Yanenko operator splitting technique is used to decouple the difficulties associated with the incompressibility constraint, the nonlinear convection term and the rigid body motion constraint. The electric force acting on a particle is calculated by integrating the Maxwell stress tensor (MST) over its surface. The MST is obtained from the electric potential, which in turn is obtained by solving the electrostatic problem. The DNS results are compared with those from the point dipole approximation. Simulations show that the accuracy of the point dipole approximation diminishes when the distance between the particles becomes smaller than the radius and also when the dielectric mismatch between the fluid and particles is greater.

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Computation of Suspensions Subjected to Electric Fields Using Multi-Image Method

A molecular-dynamics-like method is developed to simulate numerically the suspension of polarizable particles in nonconductive solvents under external electric fields. The particles experience both hydrodynamic and electrostatic interactions. The former are determined simply by using the Stokesian dynamics, assuming that the Reynolds number Re is much less than 1.0, while the latter are computed by differentiating the electrostatic energy of the suspension, itself computed from the induced particle dipoles on the particles. The multiple image method is also used to account for the interparticle force when two particles are nearly touching. Because the electrostatic energy accounts for both near- and far-field interactions, so does the force. In this paper, the motion of particles is computed for monodisperse suspensions of hard, dielectric spheres in Newtonian fluids first under a uniform electric field. Our computations are then expanded to the case where the suspension is subjected to a spatially non-uniform electric field. Our results show that the molecular-dynamics-like method presented in this paper can be used to simulate suspensions subjected to an electric field independently of whether the latter is uniform or non-uniform.

Q. Jin[1], C. Verdier, P. Singh[1], and N. Aubry[1]

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Deformation of Leukocytes Near an Uneven Wall

We use the direct numerical simulation (DNS) approach to study the motion and deformation of white blood cells, or leukocytes, in pressure driven flows in channels with smooth and uneven walls. We consider two cases: (i) when there is an adhesion force between the leukocytes and the channel wall and (ii) when the adhesion force is not present. Two composite fluid models, consisting of a membrane, cytoplasm and a nucleus, are used to describe leukocytes. The first is the composite-drop model in which the cytoplasm and the nucleus are modeled as fluids, and the second is the drop-rigid-particle model in which the cytoplasm is modeled as a fluid and the nucleus as a rigid particle. The cytoplasm is modeled as a Newtonian fluid and the nucleus in the first model is assumed to be a viscoelastic liquid. The adhesion force is computed using two adhesion force models. In the first model, the adhesion force is given by a potential that varies as the third power of the distance between the cell and the adhesive wall. In the second model, the adhesion force is given by the Bell's kinetic adhesion model. The numerical code is based on the finite element method and the level-set method is used to track the cell membrane position. In the absence of the adhesion force, the equilibrium location of a freely suspended leukocyte in a pressure driven flow in a channel is shown to depend on the ratio of the cell and plasma viscosities. The deformation of leukocytes on a layer of endothelial cells, and its dependence on the adhesion force and the wall unevenness, are analyzed. Joint work with C. Verdier (Laboratoire de Spectrométrie Physique, France), and Nadine Aubry and Pushpendra Singh (NJIT).

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Regularity of Vorticity in Viscous Incompressible Flows

Some recent results of the author about the geometrical constraints on vorticity for global regularity of 3D incompressible flows will be presented. We focus mainly on the viscous case. The results to be reported include new geometric constraints on vorticity which suppress possible development of finite-time singularity from the nonlinear vortex stretching mechanism, especially a new condition on smoothness of

the direction of vorticity in vortical region which yields regularity and a regularity condition of isotropy type on vorticity in the intensive vorticity region via a new cancellation principle.

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Presence of Chaos in a Core-Annular Flow with Surfactants

The study of core-annular flows is very important especially for oil companies. Usually, the crude is transported through water lubricated pipelinings. Surfactants are added at the interface between the two fluids. It is known that surfactants affect the surface tension. A mathematical model for an axisymmetric core-annular flow including insoluble surfactants at the interface between the two fluids was derived by the author in 2002 finding an integro-differential system of two coupled nonlinear equations. By that time the coefficient of the dispersive term was neglected. It was not possible to find chaos. In the present work, the dispersive term is considered. The numerical results show a rich behavior of the solution as the model parameters vary. The possible long time behavior of solutions includes stationary and travelling wave attractors, as well as time-periodic and chaotic attractors. Joint work with Demetrius Papageorgiou and Michael Siegel (New Jersey Institute of Technology).

Iman Kazerani and Bruce Bukiet

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Math Modeling, Statistics, and Optimization for Value Assessment in Major League Baseball

Michael Lewis's bestseller, Moneyball, highlights how analytical approaches can help baseball teams of modest means compete against wealthier teams. The values of strategies and relative influence of some performance measures on team success are discussed. Most of the book’s conclusions are based on statistical analyses. In this study, mathematical modeling, including Markov chain analysis, is used to investigate some of the claims the book makes and some questions inspired by the book.

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Transverse Instability of Interfacial Gravity-Capillary Solitary Waves of Benjamin Type

Using perturbation methods, the stability to long-wave transverse perturbations is discussed of gravity-capillary solitary waves on the interface between a infinitely deep fluid and a thin shallow fluid layer on the top with the presence of very strong interfacial tension. These solitary waves are referred as of 'Benjamin type' and explored by Calvo & Akylas (2002). The analysis is a generalization of the transverse stability analysis of gravity-capillary solitary waves for B (Bond number) $< 1/3$ on water of finite or infinite depth (Kim & Akylas 2006), and the idea is originally adopted from Tsutahara & Kataoka's analysis for the transverse instability of gravity solitary waves (2004); The leading order perturbation of the instability growth rate is expressed in terms of solitary wave solutions. It is shown that transverse instability occurs when the solitary wave energy is a decreasing function of wave speed, which is consistent with Bridges (2001).

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Googlewhack and Internet Search Result Probabilities

We study the number of search results returned by a set of words based on the results for each word when searched for individually. Our model is built on the hypothesis that these results are distributed randomly on the total number of pages indexed by the given search engine. We first analyze the special case where there is only 1 result for the two words (a Googlewhack) then apply our methods to all word sets and results. For our tests, we use a computer program to automatically mine Google and obtain the total number pages containing all words in a set. Our first set of word pairs were Googlewhack word pairs taken from Googlewhack.com. We performed a similar study for general word pairs and triplets that have more than one result.

Experiments have shown that words are not distributed randomly on webpages. In general, a given word is more likely to appear on a larger webpage than on a smaller one. The same holds for a pair of words. We built a model that uses a probability distribution for words on webpages based on Zipf's and Heaps' Laws. Simplifications allow us to compute the optimal value of the product of the Zipf and Heaps parameters based on the Googlewhack results. We developed a computational method to approximate the full model behavior and confirmed that the simplifications are valid for small numbers of search results. Our model with the new parameter fits experimental results better than our uniform model. We also show that our model and its extension to word triplets leads to better agreement with experiment than the uniform model for general word pairs and triplets.

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Thin Film Equations for Fluid Motion Driven by Surfactants

In the lubrication approximation, the motion of a thin liquid film is described by a single fourth-order partial differential equation (PDE) that models the evolution of the height of the film. When the fluid is driven by a Marangoni force generated by a distribution of insoluble surfactant, the thin film equation is coupled to an equation for the concentration of surfactant. Such films have been studied in the context of surfactant replacement therapy for the lungs of premature infants. Analysis of the PDEs and numerical simulations reveal a wide array of wave-like structures in the film, which persist when capillarity and surface diffusion are neglected. We explain the structures for the reduced system as a combination of traveling waves with discontinuities, and reveal a critical threshold at which the type of solution changes dramatically. PDE simulations and a dynamical systems approach help us understand the behavior of the solutions and how they change as higher order terms are returned to the system.

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Couplings Between Bulk Flows with Inertia and Monolayer Mesoscale Structure

Monolayer hydrodynamics are usually described in terms of a Newtonian constitutive relationship. However, this macroscopic view fails to account for small-scale co-existing phase domains, which are generally present in the monolayer and appear to have profound macroscopic effects. We consider two flow situations. First, in a flow where the monolayer is being periodically compressed and dilated, we provide direct evidence of these effects, consisting of Brewster angle microscopy images of the monolayer, space and time resolved interfacial velocity measurements and comparisons with predictions based on the Navier-Stokes equations together with the classic model for a Newtonian interface. In a second flow

system, we impart a well controlled shear-rate distribution with inertia to a monolayer consisting of coexisting phases, and study the resulting phase morphology and domain fragmentation. These evolve on distinct time scales: the viscous time associated with the viscosity in the bulk and the Marangoni stress, and the fragmentation/relaxation time associated with the phase morphology. A relationship between microstructure (line tension) and macroflow (shear rate) determining the meso length scale of the coexisting phase domains has been deduced from dimensional analysis and found to correlate well with the quantitative experimental observations. Joint work with Amir. H. Hirsa (Department of Mechanical Engineering, Rensselaer Polytechnic Institute) and Michael J. Vogel (School of Chemical and Biomolecular Engineering, Cornell University).

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Dynamics of Initial Fluctuation Decay in a Sedimenting Suspension

In a short period following the preparation of a random, sedimenting suspension in a large container, velocity fluctuations decay dramatically. During this initial period, the suspension flows as a viscous fluid of variable density. This model is formulated as a quadratic evolution equation that possess a linear subspace of stationary states (with a stable cone), a family of constants of the motion and a monotone decreasing linear functional. The dynamical consequences of this structure are explored in low dimensional models. Additionally, the effects of fluctuation decay on stratification are presented.

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Multistability in a Two-Cell Inhibitory Network with T-like Currents and Synaptic Facilitation

Networks of neurons coupled through reciprocal inhibition are ubiquitous, and are of crucial importance for maintaining rhythmic activity in such diverse systems as invertebrate central pattern generators, and the mammalian subcortical and cortical circuits.

Using analytic and computational methods, we explore the dynamics of a network of two neurons with type-I excitability, each endowed with a T-like current, and reciprocally coupled by inhibitory synapses. For sufficiently strong synaptic coupling, it is known that the T-currents allow such a circuit to maintain a stable antiphase bursting state, in which a burst in one cell causes a rebound burst in the other cell. In addition, the network we consider can maintain a low-frequency tonic state, whereby a single low-frequency spike is not sufficient to cause a postinhibitory rebound in the partner cell. Finally, we show that this two-cell network can also exhibit an irregular chaotic firing regime, corresponding to certain intermediate, partial level of T-current activation.

Without synaptic facilitation, the network is bistable in a certain parameter range, exhbiting two of the three stable activity states. We demonstrate that facilitation can significantly enlarge this multi-stability parameter range, since facilitation would serve to strenghten the synaptic coupling during a burst, increasing the entrainment of the two cells in the burst state, while allowing the synaptic currents to remain below rebound activation threshold during the low-frequency firing. Further, we show that in the presence of facilitation the irregular chaotic burst state becomes metastable, and can co-exist with the two stable periodic firing states. Finally, we find multistability of periodic bursting states with different number of spikes per burst, and show that it can be analyzed using a 1D Poincare return map. Joint work with Amitabha Bose (NJIT) and Farzan Nadim (NJIT and Department of Biological Sciences, Rutgers University, Newark).

Nebojsa Murisic and Lou Kondic

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"Octopus"-Shaped Instabilities of Evaporating Droplets

The motivation for this work stems from curious phenomena observed in applications related to semiconductor industry. In these applications, wafer surface is treated by water (DIW) and/or alcohol (IPA), which are deposited upon the surface and then left to dry. Peculiar "octopus"-shaped instabilities were noticed close to the contact line of evaporating alcohol drops. Since these satellite droplets carry significant amounts of residue which can deteriorate electrical properties of the device, we are interested in explaining why and under what circumstances these instabilities occur. We develop a mathematical model based on Navier-Stokes equations, and using lubrication limit, obtain approximate equation for evolution of droplet height. We perform linear stability analysis of this equation, and also solve it numerically in two space dimensions. The results resemble the "octopus"-shape instabilities reported in experiments. We show that the features are consequence of evaporation induced Marangoni effects within the drop.

Sai chaitanya Nudurupati, Pushpendra Singh, and Nadine Aubry

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Particle Manipulation Using Traveling Electric Fields in Microfluidic Chambers

Manipulating small sized particles such as biological, glass, polymer and carbonaceous particles suspended in a liquid is a crucial step in many microfluidic devices. One method consists in using a microfluidic chamber equipped with electrodes at the bottom and thus creating conventional dielectrophoresis based on a non-uniform electric field. In this work, we explore traveling wave dielectrophoresis generated by an electric field of spatially varying phase, which offers both particle capturing/separation and transport capabilities (avoiding pumping the fluid itself). Particles are subjected to electrostatic and hydrodynamic forces and torques that are computed solving the full equations of motion for both the fluid and the particles without any modeling (from first principles) and using a finite element scheme based on the Distributed Lagrange Multiplier (DLM) method. We consider typical microfluidic channels (MEMS devices) with electrodes placed in the bottom wall. Our results show that the transient motion and final position of the particles strongly depend on the frequency dependent complex Clausius-Mossotti factor (the mismatch between the particles and fluid electric properties), and that the hydrodynamic and electrostatic particle-particle interactions play a crucial role on the particles dynamics. These conclusions are drawn for model particles having the properties of yeast cells.

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Electrohydrodynamic Instabilities of Miscible and Immiscible Fluids in Microchannels

In recent years, there has been a growing need to understand physical phenomena at the microscale driven by a wide range of potential applications in medicine and industry. There are stark differences, however, between the macro- and microscale fluid behavior, which deserve attention. For instance, due to the low Reynolds numbers dictated by small confined geometries, good mixing is difficult to attain. One has been remedying this problem by using complex geometries, pulsed flow rates, or external fields. Another challenge of microscale fluid research has been the formation of droplets in microchannels, while controlling both the droplet size and the movement of the produced droplets in the continuous phase they are embedded in. Other issues associated with this problem are the difficulty of obtaining the desired size of droplets, as well as the dependence of the droplet size on the channel cross section.

We have performed experiments with both miscible and immiscible fluid pairs. For immiscible fluids, monodisperse droplets are formed. The drop size and formation rate are controlled by simply controlling the flow rates and the amplitude of the electric field applied across the channel. For miscible fluids, our experiments show the effectiveness of electric fields in attaining good mixing.

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Interfacial Capillary Waves in the Presence of Electric Fields

Large amplitude capillary waves on an inviscid, incompressible fluid layer of density ρ_1 bounded by a second inviscid, incompressible fluid of density ρ_2 are computed in the presence of a uniform electric field acting in a direction parallel to the undisturbed configuration. Periodic travelling waves of arbitrary amplitudes and wavelengths are calculated and the effect of the electric field is studied. The solutions extend the results of Papageorgiou and Vanden-Broeck (J. Fluid Mech. 508 (2004) 71-88; Eur. J. Appl. Math. 16 (2004) 609-623), where the case of $\rho_2=0$ was treated. Fully nonlinear solutions are computed using boundary integral equation methods. It is shown that there are both symmetric and antisymmetric waves and their characteristics are explored and compared. When there is a jump in the undisturbed horizontal velocities, the flow is susceptible to Kelvin-Helmholtz instability. It is shown analytically in the linear regime, that even in the absence of surface tension, the flow is stabilized by sufficiently large electric fields. In such situations two wave speeds are possible for given electric fields and we construct these branches numerically when the amplitudes are not infinitessimal, both in the absence and presence of surface tension.

When the wavelength of the disturbances is large compared to the undisturbed layer thickness, we use asymptotic methods to derive a novel coupled system of PDEs that govern the interfacial amplitude and leading order horizontal velocity in the layer. The equations contain the mechanisms of Kelvin-Helmholtz instability, surface tension and electric field dispersive regularization. The competition of these effects is followed into the nonlinear regime and we present numerical solutions and make comparisons with the direct simulations. Joint work with S. Grandison and J.-M. Vanden-Broeck (University of East Anglia).

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A Lab Based Mathematics Capstone Course at the University of Delaware

Just about everyone expects to find mathematics being used in the laboratory, but few of us expect to find mathematicians in the laboratory. Well, at the University of Delaware in the Department of Mathematical Sciences, finding mathematicians in the laboratory is quickly becoming a commonplace event. This was made possible with the establishment of the MEC Lab in the fall of 2002. This experimental laboratory, housed in the Department of Mathematical Sciences, takes its name from the words Modeling, Experiment, and Computation. These three words capture the hands-on philosophy of the lab; in the MEC Lab, math is meant to be experienced from every possible direction. In this poster, we explain how the MEC Lab has been placed at the center of our capstone experience for our mathematics majors. The format of the course and sample course projects are discussed. Joint work with Louis Rossi.

Muhammad M. R. Qureshi and Chao Zhu

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Crossflow Evaporating Sprays in Gas-Solid Flows: Effect of Aspect Ratio of Rectangular Nozzles

Many industrial applications of rapidly evaporating sprays in gas-solid suspensions involve the use of rectangular spray nozzles and crossflow spray injection. This paper presents a numerical study on the effects of aspect ratio of rectangular nozzles on the spray characteristics and phase interactions in such spray-gas-solid three-phase flows. The gas-solid flow is simulated via Eulerian two-fluid modeling and the spray is tracked via Lagrangian trajectory approach. Our study shows that the effect of aspect ratio has a significant impact on the penetration length and trajectories of the spray jet under the same flow conditions. The vertically oriented rectangular nozzles (i.e., nozzles with aspect ratios less than unity) have deeper penetration than the horizontally oriented ones (i.e., nozzles with aspect ratios more than unity) with the same nozzle injection area whereas the effect of aspect ratio has little impact on the spray deflection. The spray cross-section maintains the rectangular shape that expands basically following the original fan angle of the nozzle jet. The simulation also shows some strong phase interactions caused by the rapid spray evaporation, such as the formation of a dense layer of solids around the spray from the compression effect of vapor expansion and rigid wall of gas-solids flow chamber and the existence of a large diluted solids region in the vicinity of downstream of nozzle from the combined effects of evaporation and vapor cross-flow convection. Joint work with Chao Zhu.

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Coupled Reduced Model for the System Response of a Clamped Membrane Subject to a Time Dependent Electrostatic Field and a Fluid Squeeze Film

A simplified reduced model, derived from Newton's law of motion, is used to predict the system response of an electrostatically driven clamped membrane in terms of the membrane's center displacement. The coupled physics of the problem, consisting of the elasticity of the membrane, the time dependent electric field and the fluid flow, are reduced to a one-dimensional model of the force. A capacitance-based generalized equation is solve to determine the membrane deformation as a function of the applied voltage, and an energy method is used to describe the membrane stiffness. The thin fluid film of air in the gap between the fixed electrode and the membrane is modeled using the linearized compressible Reynold's gas film equation. From this, damping and stiffness coefficients are calculated, which are then used to determine the fluid force exerted by the fluid on the membrane. The response (in terms of the applied voltage and frequency) derived by using the model is found to be in good agreement with that obtained by means of a fully coupled finite element solver. The reduced model is thus a powerful tool for capturing the salient characteristics of the response, while saving vast amounts of computing time over finite element methods.

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Motion and Stability of an Annular Jet of Fluid

We examine experimentally the motion of a viscous fluid flowing down a vertical fiber. The fluid is gravitationally forced from a large tank through an orifice to create an annular jet of fluid coating the fiber. We observe that perturbations form along the jet. We find as the flow rate decreases: the distance the perturbations develop from the orifice decreases; the velocity of the perturbations traveling down the fiber decreases; and the perturbation wavelength decreases. In addition, we measure the rate at which the

perturbations grow and compare these rates to theoretical predictions for a viscous jet with no fiber. The measured growth rates are significantly less than the theoretical predictions for the no-fiber case. Thus we conclude that the fiber enhances the stability of the jet. Joint work with Justin North (Butler University).

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Immobilization and Remobilization of Gas/Liquid Interface of Rising Bubbles in a Bulk Liquid Medium Containing Surfactants

The rise velocity of a bubble rising in a stationary liquid is generally less than its theoretically predicted value. This velocity is near the rise velocity of a similar solid sphere. This behavior is attributed to the presence of surfactant impurities in the liquid. Surfactant molecules adsorb to the gas/liquid interface of the bubble. As the gas/liquid interface is mobile, the surfactants get swept to the back end of the bubble. This sweeping of the surfactants to the back end of the bubble creates a surfactant concentration gradient, which in turn creates a surface tension gradient. This surface tension gradient makes the interface rigid and hence immobile. This leads to the increase in drag on the bubble and hence the bubble rises with reduced velocity. However, if a surfactant, which has a very fast kinetics and a high solubility is used, then at low concentrations it immobilizes the interface, but at high concentrations it starts to remobilize the interface and hence are called remobilizing surfactant. There are two mechanisms by which these surfactants can remobilize the gas/liquid interface. At concentrations below the critical micelle concentration (CMC), due to the fast kinetics, the surfactants adsorb and desorbs very fast from the gas/liquid interface and thus keeps the surface concentration of the surfactant constant, thus neutralizing the effect of surface convection. At concentrations just below the CMC, to greater than the CMC, these surfactant form either a micelle zone at the rear end of the bubble or a micelle free zone at the front end of the bubble. The presence of micelle zone around the bubble leads to a constant surface concentration of surfactant on the gas/liquid interface and hence remobilizes it. We have studied this phenomenon and our experimental and numerical simulations results would be presented. Joint work with Charles Maldarelli (City College of New York) and Demetrios Papageorgiou (New Jersey Institute of Technology).

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Kinetic Theory for Neuronal Network Dynamics

We present a detailed theoretical framework for statistical descriptions of neuronal networks and derive $(1+1)$-dimensional kinetic equations, without introducing any new parameters, directly from conductance-based integrate-and-fire neuronal networks. We describe the details of derivation of our kinetic equation, proceeding from the simplest case of one excitatory neuron, to coupled networks of purely excitatory neurons, to coupled networks consisting of both excitatory and inhibitory neurons. The dimension reduction in our theory is achieved via novel moment closures. We also describe the limiting forms of our kinetic theory in various limits, such as the limit of mean-driven dynamics and the limit of infinitely fast conductances. We establish accuracy of our kinetic theory by comparing its prediction with the full simulations of the original point-neuron networks. We emphasize that our kinetic theory is dynamically accurate, i.e., it captures very well the instantaneous statistical properties of neuronal networks under time-inhomogeneous inputs.

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Instabilities and Saturation of Electrified Thin Liquid Films

The effect of normal electric fields on films lying above, or hanging from, horizontal substrates is considered. Systematic asymptotic expansions are used to derive fully nonlinear long wave model equations for the scaled interface motion and corresponding flow fields. A weakly nonlinear theory is not possible due to the absence of the shear flow generated by the gravitational force along the plate when the latter is inclined. We study the fully nonlinear equation, which in this case is asymptotically correct and is obtained at leading order. The model equation describes both overlying and hanging films - in the former case gravity is stabilizing while in the latter it is destabilizing. The numerical and theoretical analysis of the fully nonlinear evolution is complicated by the fact that the coefficients of the highest order terms (surface tension in this instance) are nonlinear. We implement a fully implicit two level numerical scheme and perform numerical experiments. We also prove global boundedness of positive periodic smooth solutions, using an appropriate energy functional. This global boundedness result is seen in all our numerical results. Through a combination of analysis and extensive numerical experiments we present evidence for global existence of positive smooth solutions. This means, in turn, that the film does not touch the wall in finite time but asymptotically at infinite time. Numerical solutions are presented to support such phenomena.

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Impact of Noise on the Onset of Vortex Breakdown

The effect of noise on the dynamical properties of a fluid flow in a bounded container is studied. The flow is modelled by the Navier-Stokes equations. The noise, which is defined by a random process with predefined statistical properties, is introduced in a physically relevant manner, via the boundary conditions. The stability of critical states of the deterministic system is analyzed via the simultaneous linearization of the system in physical and probability spaces. The resulting equations form a stochastic system, which depend on the deterministic critical state and on a Wiener process whose auto-correlation function is directly connected to the type of noise as well as on the critical state itself. The behavior of the stochastically forced system around critical states is explored numerically. Preliminary results will be presented. Joint work with Juan Lopez (Arizona State University) and Francisco Marques (Universidad de Catalunya, Spain).

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Formation of Ligaments on a Sheared Liquid-Gas Interface

A strong gas flow parallel to a liquid-gas interface tends to shatter it into small droplets. This process occurs naturally when stong winds over the ocean generate sea-spray, and by design in combustion technology (and other applications) which depend on the atomization of liquids into droplets. Current understanding of the mechanisms leading to atomization rests in part on various theoretical approaches and in part on experimental observation, while there are relatively few numerical studies. We describe here work that focuses on the influence of viscosity on ligament formation and droplet sizes. The emphasis is on the high speed regime, very far from the Savart-Plateau-Rayleigh jet instability, which leads to droplets of roughly the size of the jet. At these higher Weber and Reynolds numbers, instabilities of much shorter length scale develop and the process is more complex, involving turbulence upstream of the injector (as argued by Faeth), instabilities near the injector due to the shearing motion of the liquid and the gas,

ligament and droplet formation and dispersion in an initially dense spray. The region near the nozzle, where spray formation is initiated by the growth of surface instabilities is thus especially interesting. We present numerical simulations of two-phase liquid-gas sheared layers, with the objective of studying atomization. The Navier-Stokes equations for two-dimensional incompressible flow are solved in a periodic domain. A Volume of Fluid (VOF) method is used to track the interface. The value of the density ratio is kept near ten. The calculations show good agreement with a fully viscous Orr-Sommerfeld linear theory over several orders of magnitude of interface growth. The nonlinear development shows the growth of finger-like structures, or ligaments, and the detachment of droplets. The effect of the Weber and Reynolds numbers, the boundary layer width and the initial perturbation amplitude are discussed through a number of typical cases. An initially inverted liquid boundary layer, the typical case in laboratory experiments, is shown to lead more readily to ligaments bending upwards and is thus more likely to produce droplets. Joint work with S. Zaleski, E. Lopez-Pages, J. Li, and T. Boeck.

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Multimodal Regimes in Systems Close to a Degenerate Andronov-Hopf Bifurcation

A one-parameter family of three-dimensional flows near an Andronov-Hopf bifurcation (AHB) is investigated in this work. We identify conditions on the global vector field, which yield a rich family of multimodal orbits passing close to a weakly unstable saddle-focus and provide a detailed asymptotic description of these orbits near the saddle-focus. The analysis covers both cases of sub- and supercritical AHB. If the AHB is subcritical, it is accompanied with appearance of the multimodal orbits, which consist of long series of small amplitude nearly harmonic oscillations separated by large amplitude spikes. We analyze the dependence of the interspike intervals (which can be extremely long) on the control parameters present in the system. In particular, we show that the interspike intervals increase significantly as the boundary between the regions of sub- and supercritical AHB bifurcation is approached from the subcritical region. Furthermore, we identify a window of complex dynamics near the boundary between the regions of sub- and supercritical AHB. This work is motivated by the numerical results for a finite-dimensional approximation of a free boundary problem modeling solid fuel combustion.

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Interesting Dynamics of Viscous Drops in a Straining Flow with Rotation

Fluid flows with low Reynolds numbers have a wide range of applications. In this poster interesting dynamics of emulsion of viscous drops in a straining flow with rotation will be presented. Emulsion drops with high viscosity are found to behave in hysteretic fashions, and chaotic dynamics may be found as a result. This has direct application to flow control in micro-fluidic devices. Joint work with Jerzy Blawzdziewicz and Vittorio Cristini.

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Novel Dynamics of Elastic Filament in Stokes Flow

Interesting dynamics of a slender, inextensible filament in Stokes flows is found in simulations using a hybrid numerical scheme that combines the Hasimoto transformation with the slender body formulation.

From the simulations of a filament in periodic flows we see the filament flexes, swims, and ties knots onto itself due to the elasticity. The numerical scheme specifically designed to accurately capture these complex dynamics will be summarized. Novel filament dynamics in various different flow configurations will be presented. Joint work with Mike Shelley.

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Effect of Surfactant and Surfactant Solubility on the Breakup of a Bubble via Thread Formation

The influence of surfactant on the breakup of a bubble in a quiescent viscous surrounding is studied by a combination of direct numerical simulation and the solution of a longwave asymptotic model. The direct numerical simulations describe breakup of an inviscid bubble, while the effects of small but nonzero interior viscosity are readily included in the longwave model within the confines of the Stokes flow limit.

The direct numerical simulations use a specific but representative and realizable initial bubble shape to compare the evolution and breakup of: a clean or surfactant-free bubble, a bubble coated with insoluble surfactant, and a bubble coated with soluble surfactant. The main feature of the evolution in the presence of surfactant is the interruption of bubble breakup by formation of a slender quasisteady thread of the interior fluid.

The longwave asymptotic model, for a slender thread with periodic boundary conditions, explains the principal mechanism of thread formation and confirms, for example, the minor role played by the Marangoni stress. The large-time evolution of the thread and precise location of its breakup are, however, influenced by Marangoni stress, surface diffusion of surfactant, and surfactant solubility. Joint work with J. Li (University of Cambridge), and M. Hameed and C. Maldarelli (City College, CUNY).