PLENARY SPEAKERS

Grigory Barenblatt

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Incomplete Similarity in Continuum Mechanics

The general concept of incomplete similarity will be introduced and discussed. The connection of this concept with dimensional analysis, intermediate asymptotics, and renormalization groups will be presented. The examples of scaling laws having the property of incomplete similarity will be presented, in particular from turbulence (turbulence in pipes, boundary layers, and wall-jets), fatigue of metals and fracture.

John Hinch

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Collapse of a Column of Grains

Motivated by landslides, the collapse of a tall column of grains onto a horizontal plane is studied, with particular interest in the distance the grains run out. Previous experiments find simple scaling laws in axisymmetric and 2D geometries which defy simple explanation. Numerical simulations of several thousands of discrete rigid 2D grains recover many of the detailed results of the 2D experiments. Additionally the simulations reveal a near free-fall of the grains within the column until they are close to the bottom, where they turn into a thin flowing horizontal layer. Idealising this picture, shallow-water equations with Coulombic basal friction are solved for the horizontal flowing layer, with grains added over time as `rain' with no horizontal momentum over the base of the initial column. This model recovers the experimental scalings of the runout distance in both 2D and axisymmetric geometry. There remains, however, the task of explaining the simple scaling laws. The solutions of the shallow-water equations show three distinct phases:- (i) an initial quitting the base of the column, where the different geometries produce different results, (ii) a propagating wave, which is responsible for half the runout, and (iii) a final deceleration. While each of the phases have different scalings, their sum results in the simple form found in the experiments over the range where experiments have been performed.

Thomas Hou

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The Interplay between Local Geometric Properties and the Global Regularity of the 3D Incompressible Euler Equations

Whether the 3D incompressible Euler equation can develop a finite time singularity from smooth initial data has been an outstanding open problem. Here we review some existing computational and theoretical work on possible finite blow-up of the 3D Euler equation. We show that there is a sharp relationship between the geometric properties of the vortex filament and the maximum vortex stretching. By exploring this local geometric property of the vorticity field, we have obtained a global existence of the 3D incompressible Euler equations provided that the unit vorticity vector and the velocity field have certain mild regularity properties in a very localized region containing the maximum vorticity. Further, we perform large scale computations of the 3D Euler equations to re-examine the alleged finite-time blowup of the two antiparallel vortex tubes, which has been considered as one of the most attractive candidates for a finite-time blowup of the 3D Euler equations. Our numerical studies indicate that the maximum vorticity does not grow faster than double exponential in time. The velocity field is bounded throughout the computations. The vortex lines near the region of the maximum vorticity are relatively straight. This local geometric regularity of vortex lines seems to be responsible for the dynamic depletion of vortex stretching. Finally, we will present a new class of solutions for the 3D Euler and Navier-Stokes equations, which exhibits a very interesting dynamic growth property but they have global existence for all times.

Charles S. Peskin

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Cardiac Mechanics and Electrophysiology by the Immersed Boundary Method

The fluid dynamics of blood in the cardiac chambers is inherently linked to the elastic dynamics of the heart valves and also to the muscular dynamics (an active kind of elasticity) of the heart walls. Coordination and control of the heartbeat requires, moreover, an electrical system to signal the heart muscle when to contract and when to relax. This lecture describes how the immersed boundary (IB) method can be used to give a unified mathematical formulation to the above electromechanical problem, as well as a unified computational framework for its numerical solution. Results will be shown as computer-generated animations. Joint work with David Mcqueen and Boyce Griffith.

MINISYMPOSIUM SPEAKERS

Triantafyllos R. Akylas

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On Gravity-Capillary Lumps and Related Problems

Fully localized three-dimensional solitary waves, commonly referred to as 'lumps', have received far less attention than two-dimensional solitary waves in dispersive wave systems. Most studies have considered the long-wave limit, where lumps can be found when the long-wave speed is a minimum of the phase speed and are described by the Kadomtsev-Petviashvili (KP) equation. In the water-wave problem, in particular, lumps of the KP type are possible only in the strong-surface-tension regime (Bond number, $B > 1/3$), a condition that limits the water depth to a few mm.

We report on a new class of lumps that is possible under less restrictive physical conditions. Rather than long waves, these lumps bifurcate from infinitesimal sinusoidal waves of finite wavenumber at an extremum of the phase speed. As the group and phase velocities are equal there, small-amplitude lumps resemble fully localized wavepackets with envelope and crests moving at the same speed, and the wave envelope along with the induced mean-flow component are governed by a coupled Davey-Stewartson equation system of elliptic-elliptic type. The lump profiles feature algebraically decaying tails at infinity owing to this mean flow.

In the case of water waves, lumps of the wavepacket type are possible when both gravity and surface tension are present on water of finite or infinite depth for $B < 1/3$. Moreover, a linear stability analysis of the gravity-capillary solitary waves, that also bifurcate at the minimum gravity-capillary phase speed, reveals that they are always unstable to transverse perturbations, suggesting a mechanism for the generation of lumps.

This generation mechanism is further explored in the context of the two-dimensional Benjamin (2-DB) equation, a generalization to two horizontal spatial dimensions of the model equation derived by T.B. Benjamin for uni-directional small-amplitude long interfacial waves in a two-fluid system with strong interfacial tension. The 2-DB equation admits solitary waves and lumps of the wavepacket type analogous to those bifurcating at the minimum gravity-capillary phase speed in the water-wave problem. Based on unsteady numerical simulations, it is demonstrated that the transverse instability of solitary waves of the 2-DB equation results in the formation of lumps, which propagate stably and are thus expected to be the asymptotic states of the initial-value problem for fully locally confined initial conditions. This is joint work with Boguk Kim.

Nadine Aubry

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Microfluidic Mixing in Channels of Simple Geometry

Many microfluidic applications require mixing for rapid reaction processes although the latter has remained a challenge. While complex geometries have been the focus of many other works, this research has concentrated on finding solutions for channels of simple geometries. The first solution uses out of phase time pulsing in the two inlets bringing the two fluids together. A detailed analysis shows that stretching and folding of material lines and chaotic advection are responsible for the mixing for certain parameter values. The second solution induces mixing by using the interfacial instability between the two fluids to be mixed, which occurs when two fluids with different electric properties are subjected to an electric field perpendicular to the interface. The linear stability analysis is carried out taking into account the fully coupled fluid dynamics/electric problem. The latter technique, when applied to miscible fluids, has been shown to generate efficient mixing and, when applied to immiscible liquids, has been demonstrated to be a controllable monodispersed droplet generator. In all cases, both analytical and experimental results will be shown. This work was performed in collaboration with I. Glasgow, A. Goullet, A. Ould El Moctar, O. Ozen, D. Papageorgiou, and P. Petropoulos.

Neil Balmforth

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Instability in Flow through Elastic Conduits and Volcanic Tremor

The stability of fluid flow through a narrow conduit with elastic walls is explored, treating the fluid as incompressible and viscous, and the walls as semi-infinite, linear Hookean solids. Instabilities analogous to roll waves occur in this system; we map out the physical regime in which they are excited. For elastic wavespeeds much higher than the fluid speed, a critical Reynolds number is required for instability. However, that critical value depends linearly on wavenumber, and so can be made arbitrarily small for long waves. For smaller elastic wavespeeds, the critical Reynolds number is reduced still further, and Rayleigh waves can be destabilized by the fluid even at zero Reynolds number. A brief discussion is given of the nonlinear dynamics of the instabilities for large elastic wavespeed, and the significance of the results to the phenomenon of volcanic tremor is presented.

Andrew J. Bernoff

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Domain Relaxation in Polymer Langmuir Layers

A Langmuir layer is a molecularly thin layer of a lipid or polymer on the surface of a fluid. Depending on density, the layer can act as a quasi-two-dimensional gas, liquid, liquid crystal, or solid. For a localized fluid phase, intermolecular forces create a line tension that tends to minimize the domain perimeter; the driving of the line tension is opposed by the drag of the viscous subfluid. In this talk, I will report on two experimental, theoretical, and computational studies of this relaxation process in which we try to estimate the magnitude of the line tension. In the first study, we consider the stretching (by a transient stagnation flow) of a circular domain into a slender cigar-shaped region. The domain relaxes first into a dumbbell shape with two roughly circular reservoirs connected by a narrow tether. This shape eventually relaxes to the minimum energy configuration of a circular domain. Amazingly, the tether is never observed to rupture, even when it is more than a hundred times as long as it is thin. We simulate this relaxation numerically via a boundary integral formulation; comparisons with experiment allow us to deduce the line tension. In the second study, we consider the collapse of a Langmuir gas bubble surrounded by a fluid domain. We find a similarity solution for the bubble collapse, and determine the rate of contraction by formulating and solving a pair of dual integral equations. Comparison with experiment again allows us to estimate the line tension in the system. Joint work with James C. Alexander (Department of Mathematics, Case Western Reserve University), Elizabeth Mann (Department of Physics, Kent State University), J. Adin Mann, Jr. (Department of Chemical Engineering, Case Western Reserve University), Jacob M. Pugh (Department of Physics, Harvey Mudd College), and Lu Zou (Department of Physics, Kent State University).

Jerzy Blawzdziewicz

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Stepwise Drainage of Thin Liquid Films Stabilized by Colloidal Particles

Particle-stabilized films do not drain continuously but, instead, they undergo a series of stepwise transitions between states of different thickness. This behavior stems from the oscillatory structural forces produced by the particles present in the film. Both normal and tangential components of these forces are shown to be essential for description of equilibrium and non-equilibrium film properties. In our approach the film is described using a quasi-two dimensional thermodynamic formalism, in which the key intensive parameters are the normal pressure and effective film tension. The latter quantity is obtained from the difference between the normal and tangential pressure components. For a film stabilized by a hard-sphere suspension, we have evaluated the thermodynamic equations of state, and determined conditions for the coexistence of film phases of different thickness. Our formalism is also generalized for films out of equilibrium. In the linear-response regime, the film dynamics is characterized by the shear and extension viscosity coefficients, and by two kinetic coefficients relating the particle flux to the gradients of the normal osmotic pressure and particle chemical potential. For a hard-sphere suspension, these coefficients have been evaluated using a hydrodynamic multipolar-expansion method combined with a flow reflection technique. Joint work with E. Wajnryb.

Oliver Buhler

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The Likely Shape of Large Waves

There is often a special interest in the structure of large-amplitude waves in geophysical fluid dynamics and elsewhere. For instance, large ocean surface waves (the "rogue waves") can be a shipping hazard, and large internal ocean waves can provide important deep-ocean mixing by their turbulent breaking. This talk presents some classical fluid dynamics about the structure of large waves together with some current work on the most likely shape of large random waves based on large deviation theory.

Darren Crowdy

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Cusps, Threads, and Steady Pinchoff in Free Surface Stokes Flow

This talk will survey the use of complex variable methods in free surface Stokes flows. In particular, we return to a classic problem considered by Jeong and Moffatt (JFM, 1992) on the deformation of an infinite free surface by two submerged rollers and the observed formation of a cuspidal structure. Jeong and Moffatt constructed an exact solution of a model of this experimental arrangement using complex variable methods and demonstrated that the curvature of the interface grows exponentially with the capillary number. It will be pointed out that, with slight generalization of how the driving mechanism is modelled, the same mathematical system admits a variety of other free surface profiles including thin extended threads of fluid penetrating the viscous bath, as well as steady capillary bubble pinchoff.

Linda Cummings

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Fluid Flow Problems in Rotating Bioreactors

Bioreactors are used in tissue engineering applications to grow functional tissue in vitro. A set-up of particular interest is the rotating bioreactor, which is essentially a cylinder filled with a nutrient solution into which tissue cells, often first "seeded" onto an appropriate scaffold, are placed. The tissue cells proliferate in response to the supplied nutrient, and the whole assembly is rotated about the cylinder's axis.

We consider an idealised mathematical model of the rotating bioreactor, in which its aspect ratio is asymptotically small, an assumption that leads to novel "Hele-Shaw" problems. Two problems of interest are studied: (i) a shape stability problem (irregularly-shaped tissue constructs are observed by our experimental collaborators); and (ii) the motion of a tissue construct within the nutrient solution under fluid-dynamical forces. In the latter case, the fluid flow and construct trajectory may be determined analytically, and hence the convection-diffusion problem governing nutrient transport throughout the bioreactor may be solved, and the subsequent tissue growth-rate determined.

Robert Ecke

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Experiments in Two-Dimensional Turbulence

I will describe experiments in physical realizations of 2D turbulence including soap film flows and thin stratified layers. Using these systems, I will show how one can achieve a systematic and quantitative understanding of the physics of 2D turbulence. The critical part of our work has been the close synthesis of theory and numerical simulations with the experimental results. Using a filter approach similar to the one employed for large-eddy simulations, we obtain details of the energy and enstrophy transfer mechanisms in 2D turbulence. I will also describe our Lagrangian measurements in 2D flows and the insight these give into the turbulent state.

A. S. Thanasis Fokas

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On a New Nonlocal Formulation of Water Waves

The classical equations of water waves are reformulated as a system of two equations, one of which is an explicit nonlocal equation, for the wave height and for the velocity potential evaluated on the free surface. Evaluation of the velocity potential as a function of the depth is not required in order to calculate the wave height and the velocity potential on the free surface. The nonlocal system yields integral relations related to mass and center of mass, and is shown to reduce to known asymptotic limits in shallow and deep water. Included in these asymptotic reductions are the Boussinesq, Benney-Luke, and nonlinear Schroedinger equations. Two-dimensional lumps with sufficient surface tension are obtained numerically. The extension of this nonlocal formulation to the case of variable bottom also is presented.

Don Gaver

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The Importance of Surfactant Physicochemical Hydrodynamics in Pulmonary Atelectrauma

The lung consists of many bifurcating airways that terminate with alveoli, the site of gas exchange. Pulmonary surfactant, a protein-phospholipid mixture, is released from alveolar type-II pneumocytes, and has the primary function of dynamically reducing the surface tension of the lining fluid. Without proper surfactant molecular function, the lung is micro-mechanically unstable, leading to airway closure and insufficient gas exchange. This presentation will focus on theoretical and in-vitro experimental investigations of pulmonary airway reopening. We present our observations of cellular damage to epithelial cells as a result of the migration of a bubble progressing along the cell surface and relate the observed damage to stress magnitudes predicted from computational investigations of model systems. Finally, we experimentally and computationally investigate novel forms of mechanical ventilation that may protect the lung from atelectrauma by taking specific advantage of the dynamic surface tension behavior of pulmonary surfactant. Funded by NASA NAG3-2734 and NIH P20 EB001432-01.

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Braided Rivers and Stability Theory

Sediment carrying rivers support various modes of instability leading to complex shapes for the bedform. Perhaps the most complex situation is when the river develops a braided pattern with many channels and islands forming and disappearing as time moves on. We show how many key features of braided rivers can be described by ideas from hydrodynamic stability theory. Some new evolution equations are derived and some highly nonlinear states of a type not usually accessible in hydrodynamic stability theory are described and comparisons with observations suggest the models used capture most of the observed phenomena.

Karl Helfrich
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The Effects of Rotation on Strongly Nonlinear Internal Waves

Oceanic observations of internal solitary-like waves often show that they are highly nonlinear. These large waves are frequently the product of disintegration of the internal tide and may propagate large distances. As a consequence, the role of planetary rotation on their emergence and evolution should not be ignored. To address these issues, the fully nonlinear, weakly-dispersive two-layer model of Miyata (1988) and Choi and Camassa (1999) is extended to include rotation. This model then is used to explore two related problems of large amplitude internal solitary wave evolution. In the first, radiation damping of a large solitary wave is studied. Numerical solutions show that a radiated long inertia-gravity (IG) wave can itself steepen to produce a new solitary wave. This process may lead to a nearly periodic decay and re-emergence of a primary solitary-like wave. In the second problem, the disintegration of an initial long sinusoidal IG wave (i.e., the internal tide) is discussed. It is shown that the presence of fully nonlinear, hydrostatic IG wave solutions can prevent the full disintegration of the initial IG wave into a packet of nonlinear solitary-like waves. The degree of disintegration is controlled by the proximity (in terms of energy) of the initial state to an underlying nonlinear IG wave. The application of the model to the ocean is discussed.

Diane Henderson
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Stability, Instability, and Stability in Deep-Water Surface Waves

Here we report experiments on permanent form gravity waves on deep water propagating in both one and two horizontal dimensions. We find that moderate amplitude, bi-periodic patterns are ``stable'' within the length of our wave basin. This result is surprising in light of classic instability results (the Benjamin-Feir instability) for deep-water waves. And large amplitude experiments do show evidence of what appears to be the Benjamin-Feir instability. However, recent numerical results by Fuhrman & Madsen (2006) provide a different explanation. Our further experiments show that their explanation is correct and the patterns are indeed ``stable''. To explain the unexpected persistence of these patterns mathematically, we reconsider the stability of a uniform wavetrain using the nonlinear Schroedinger (NLS) equation modified to include linear damping. We prove that the presence of damping, no matter how small, stabilizes (with linear and nonlinear stability) the uniform wavetrain solution. The predicted evolutions are in excellent agreement with our experiments. These stability results then are extended to the case of a permanent form solution of coupled NLS equations that model wave patterns.

Sam Howison
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Fast Flows of Thin Layers

I shall talk about models for `hypercritical' shallow water flows, in which the Froude number is large. In such systems, the shallow water system is nearly degenerate. I shall describe the use of `delta shocks' and `delta sinks' in modelling singularities in such flows, and their application in large Froude number flows over sloping bases. In particular, I shall describe a model for a `mass tube' forming at the edge of such a sheet of fluid; this is easily visualised when a fast jet of water hits a vertical wall. The talk describes joint work with Carina Edwards, John Ockendon, and Hilary Ockendon.

Huaxiong Huang
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Moisture Transport and Diffusive Instability during Bread Baking

In this talk, we discuss two related multiphase models for simultaneous heat and mass transfer process during bread baking. Our main objective is to provide an explanation and a remedy to the observed erroneous and/or divergent results associated with the instantaneous phase change model used in the literature. We propose a reaction-diffusion model based on the Hertz-Knudsen equation, where the phase change is not instantaneous but determined by an evaporation/condensation rate. A splitting scheme is designed so that a relation between these two models can be established and the non-intuitive numerical instability associated with the instantaneous phase change model can be identified and eliminated through the reaction-diffusion model. The evaporation/condensation rate is estimated from balancing these two models and reasonable and consistent results are produced by using the estimated rate. For the evaporation/condensation rate beyond the estimated value oscillation and an interesting ``fingering'' phenomenon with multiple regions of dry and two-phase zones is observed. We show that these are caused by an instability intrinsic to the model (which we call diffusive instability) and the effect of the diffusive instability to the bread-baking simulation also is explained through a linear stability analysis and supported by numerical tests. This is joint work with P. Lin and W. Zhou.

Oliver Jensen
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The Spreading and Stability of a Surfactant-Laden Drop on a Prewetted Substrate.

A viscous drop laden with insoluble surfactant, spreading on a plane surface that is prewetted with a thin liquid film, can exhibit striking fingering instabilities. This instability has been captured recently via numerical simulations using a model based on lubrication theory (Warner et al. (2004) J. Fluid Mech. 510, 169-200). I will describe a complementary approach to the problem, exploiting the multi-region asymptotic structure of unsteady spreading solutions, making it possible to predict spreading rates and to characterize the mechanism whereby perturbations to the drop's contact line become unstable. Fundamental to the dynamics is the formation of an ultra-thin film ahead of the drop's contact line, which connects a contact line region (described by a modified version of Tanner's law) to a rarefaction wave. The analysis shows how initial conditions and transient dynamics have a long-lived influence on late-time flow structures, spreading rates, and contact-line stability.

Chung K. Law

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Experimental Perspectives on the Formulation of Laminar Flame Theory

There has been a tremendous advance in combustion science since the mid-1970s. An area that has seen the most spectacular achievement is the formulation of the theory of laminar flames, accomplished through a strong interplay between mathematical analysis and experimentation. The talk will address several aspects of such interplays, including the determination of laminar flame speeds, effects of aerodynamic stretch, mixture nonequidiffusion, and heat loss on flame extinction and instability including spiral waves, and the attempt to achieve quantitative predictability. The emphasis will be on the physical interpretation of the mathematical results.

Jian-Guo Liu

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Stability and Convergence of Efficient Navier-Stokes Solvers via a Commutator Estimate

For strong solutions of the incompressible Navier-Stokes equations in bounded domains with velocity specified at the boundary, we establish the unconditional stability and convergence of discretization schemes that decouple the updates of pressure and velocity through explicit time-stepping for pressure. These schemes require no solution of stationary Stokes systems, nor any compatibility between velocity and pressure spaces to ensure an inf-sup condition, and are representative of a class of highly efficient computational methods that have recently emerged. The proofs are simple, based upon a new, sharp estimate for the commutator of the Laplacian and Helmholtz projection operators. This allows us to treat an unconstrained formulation of the Navier-Stokes equations as a perturbed diffusion equation. This is joint work with Bob Pego and Jie Liu.

Paul Milewski

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Stability, Breaking Waves, and Mixing in Stratified Flows

The shallow water or hydraulic limit for wave propagation and breaking is well understood. At shocks, conservation of mass and momentum completely define the hydraulic jump, and the energy equation gives, a posteriori, the rate at which "internal" energy (mostly in the form of small scale turbulence) is generated. For the case of two-layer shallow water, even the evolution of smooth solutions was not completely understood since there is the possibility for the system to be elliptic, a remnant of the Kelvin-Helmholtz instability.

We first show that these flows are nonlinearly stable: smooth solutions never cross into the elliptic domain. Then, we consider shocks at the interface of miscible fluids - a problem of geophysical importance in the atmosphere and ocean. We require an additional postulate that would yield the mixing rate at the shock: one can imagine that the energy dissipated at the shock now can flow both into small scale turbulence and into mixing the fluid, but the partition between these sinks of macroscopic energy is unknown. We discuss two possibilities for deriving the additional constraint on the problem: kinematics and entropy maximization. We show that these are in fact equivalent and yield upper bounds on the mixing rates.

James Quirk

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Document Engineering and Computational Fluid Dynamics

With the development of electronic-publishing formats, like PDF, a new field known as document engineering is emerging that views a document more as a computer program than as a piece of static text. This paradigm, applied to computational fluid dynamics, raises the possibility of having self-substantiating, journal articles. These are articles where the interested reader can try out the reported work first hand, as the required material (computer code, input data, etc.) is embedded within the article. This talk will explore the scientific motivations for harnessing document engineering in this fashion, and concrete examples will be shown to demonstrate the practicability of using PDF for archiving fluid-flow simulations.

Weiqing Ren

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The Moving Contact Line Problem

The moving contact line problem is a classical problem in fluid mechanics. The difficulty stems from the fact that the classical continuum theory with no-slip boundary condition predicts a non-physical singularity at the contact line with infinite rate of energy dissipation. Many modified continuum models are then proposed to overcome this difficulty. They all succeed in removing the singularity, but they leave behind the question: which one of these models is a good description of the microscopic physics near the contact line region? We will review the results obtained using continuum theory, molecular dynamics, and the more recent multiscale techniques. We also will discuss how these techniques can be combined to give us a better understanding of the fundamental physics of the moving contact line and formulate simple and effective models. Joint work with Weinan E (Princeton University).

David Rumschitzki

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A Model for Macromolecular Transport within Heart Valves

It is well known that under conditions of high blood cholesterol concentrations the leaflets that comprise the heart valves, e.g., the aortic valve, accumulate cholesterol. The leaflet has two endothelial layers, one on each side or aspect, and an interposed region between them. The cholesterol accumulates in the interposed region, close to the endothelium. Tompkins and coworkers injected squirrel monkeys with labeled LDL and sacrificed the monkeys 30 minutes later. They harvested the monkey’s valve leaflets and determined the LDL concentration as a function of depth into the valve for various sections taken perpendicular to the plane of the endothelia. They found a very large variation both in the magnitude and the shapes of these LDL concentration profiles. They used a simple, one-dimensional diffusion model with linear membrane crossing boundary conditions to model each of these profiles and, although the curves resembled the data, each section required very different values of the two membrane constants and the diffusivity in the interposed layer. The source of this large spread in parameter values, both between animals and even between different sections taken from the same animal and even from the same leaflet has remained a mystery.

In this paper, we report on experiments that question the assumptions of this early model. We construct and solve a detailed model based on these ideas. We consider a valve leaflet with two endothelia, with focal, rather than uniform macromolecular leaks, convective transport, and a layered interposed region. The model has a single set of parameters, most of which are measured, and the remaining three are fit once and for all from one experiment. The model's results then quantitatively explain all of Tompkins' curves, where the only thing that we vary between curves is the distance of the section examined from a localized leak. It also explains other experimental results in the literature, again without changing parameters. Joint work with Zhongqing Zeng and Yongyi Yin (CCNY) and Kung-Ming Jan (Columbia College of Physicians and Surgeons).

Mark Short

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Detonation Stability and Structure: A New Formulation for Condensed Phase Explosives

In gaseous mixtures, detonation waves are inherently unstable. The nonlinear manifestation of the instability are triple shock interaction points that propagate along the lead shock wave. Regions of large vorticity in the vicinity of such triple points lead to spectacular fish-scale patterns etched from soot-covered foil. In condensed phase materials, comparatively little is know about detonation wave structure and stability.

In this talk, a new formulation of the linear stability analysis of planar steady detonation waves for multi-step reaction mechanisms and an arbitrary (incomplete) equation of state will be presented. This general formulation is the first to properly (in a mathematical sense) address the stability analysis of a steady Chapman-Jouguet detonation, which is characterized by a sonic point downstream of the lead shock. The discussion will focus on recent research in the modeling of detonation wave dynamics and stability in condensed-phase systems using continuum level equation of state modeling.

Shu-Ming Sun

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2D and 3D Surface Waves in Water with Surface Tension

The talk discusses recent development on the existence theory of two-dimensional (2D) and three-dimensional (3D) waves on free surface of water with surface tension using the fully nonlinear governing equations. It will be shown that 2D solitary waves exist for large surface tension while there are no such waves for some small surface tension. In the case of large surface tension, there exist 3D solitary waves and 3D non-solitary waves bifurcating from 2D solitary waves. The stability of 2D solitary waves also will be discussed.

Jean-Luc Thiffeault

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Chaotic Advection in Thin Films?

We consider the steady gravity-driven flow of a thin layer of fluid over a substrate with large bumps. Even though the flow is steady, there is a small vertical component of the velocity field that makes it three-dimensional in a nontrivial manner, opening the possibility for chaotic advection. For instance, is it possible to induce large lateral excursions of fluid particles by merely tweaking the shape of the substrate?

This would lead a small blob of dye to spread rapidly as it is advected by the flow, in a manner not possible in a nonchaotic regime.

Anna-Karin Tornberg

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Fluid-Structure Interactions: Suspensions of Fibers and Beyond

One important class of fluid-structure interactions concerns flows with microscopic immersed objects. These are particularly difficult to simulate as the fluid micro-structure may be highly flexible, or have large aspect ratios, or both. Understanding their dynamics is important to both engineering (dynamics of complex fluids, micro-fluidic mixing) and to biology (characterizing the biophysical properties of bio-polymers, locomotion of micro-organisms). Accurate numerical simulations can provide invaluable detailed data for these very complex systems.

We have developed computational methods both for flexible and rigid slender fibers immersed in an incompressible fluid. In this talk, we give some details for the case of microscopic rigid fibers, that sediment due to gravity. Our numerical algorithm is based on a non-local slender body approximation that yields a system of coupled integral equations, relating the forces exerted on the fibers to their velocities, taking into account the hydrodynamic interactions of the fluid and the fibers. In the case of rigid fibers, the system is closed by imposing the constraints of rigid body motions. The fact that the fibers are straight also have been exploited further in the design of the numerical method. We briefly discuss the extension to active suspensions, with self-propelling objects.

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Gravity Capillary Waves on Electrified Fluid Sheets

Nonlinear waves propagating at the interface between two fluids are considered. The effects of gravity and surface tension are taken into account and a vertical electric field is applied. The fully nonlinear problem is solved by boundary integral equation methods. Various asymptotic solutions are derived. In particular, it is shown that the problem can be modelled by a fifth order Benjamin-Ono-Korteweg de Vries equation.

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Coarsening Dynamics of Thin Fluid Films

The study of instabilities of thin fluid films on solid surfaces is of great importance in understanding coating flows. These instabilities lead to rupture, the formation of dry spots, and further morphological changes that promote non-uniformity of coatings; these behaviors in unstable thin films are generally called dewetting. To account for these effects, lubrication models of fluid flow must incorporate terms describing the influence of material properties of the solid and fluid. The resulting nonlinear PDE can accurately reproduce the complex physical pattern formation observed in experiments. Following initial transients, the film breaks up into an array of droplets. The evolution of this system can be represented in terms of coupled ODEs for the masses and positions of the droplets. Regimes where droplet coarsening by each of two mechanisms (collision and collapse) are identified, and power laws for the statistics of the coarsening processes are explained. This is joint work with Karl Glasner (University of Arizona).

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Thermal Instability in Drawing Viscous Threads

We consider the stretching of a thin viscous thread, whose viscosity depends on temperature, that is heated by a radiative heat source. The thread is fed into an apparatus with a fixed speed and stretched by imposing a higher pulling speed at a fixed downstream location. We show that thermal effects lead to the surprising result that steady states exist for which the force required to stretch the thread can decrease when the pulling speed is increased. By considering the nature of the solutions, we show that a simple physical mechanism underlies this counterintuitive behavior. We study the stability of steady-state solutions and show that a complicated sequence of bifurcations can arise. In particular, both oscillatory and non-oscillatory instabilities can occur in small isolated windows of the imposed pulling speed.

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Free Solid Boundaries in Thermal Convection

Thermal convection has come to be regarded as one of the prototype systems of dynamical systems. It often consists of a fluid confined within a rigid box that is heated at the bottom and cooled at the top. Our experimental studies explore the intriguing phenomena when its rigid boundary is partly replaced either by a freely moving, thermally opaque "floater" or by a collection of free-rolling spheres. We identify within our table-top experiments several dynamical states, ranging from oscillation to localization to intermittency. A phenomenological, low-dimensional model seems to reproduce most of the experimental results. Through our on-going experiments, we further seek their possible implications in geophysical processes such as continental drift.

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Diffusion and Mixing in Fluid Flow

We study enhancement of diffusive mixing on a compact manifold by a fast incompressible flow. We obtain a sharp description of the class of flows that make the deviation of the solution from its average arbitrarily small in an arbitrarily short time, provided the flow amplitude is large enough. The necessary and sufficient condition on such flows is expressed naturally in terms of the spectral properties of the dynamical system associated with the flow. In particular, we find that weakly mixing flows always enhance dissipation in this sense. This work also has applications to quenching in reaction-diffusion equations.