Computational Method to Evaluate Ankle Postural Stiffness with Ground Reaction Forces

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Abstract

We examine an existing method for evaluating postural sway based on force plate technology. Through an improved mathematical model of postural dynamics, we propose a new method which better evaluates postural sway, and in addition, computes ankle moment and ankle postural stiffness directly from the measured ground reaction force. An example is detailed that demonstrates the utility of this approach. The proposed method does not involve filtering or numerical integration, and takes into account the platform inclination. Results from normal subjects show a linear relation between the ankle moment and the sway angle during quiet standing.

Key Words: Postural Stability, Quiet Standing, and Ankle Postural Stiffness

Introduction

Postural control is the control needed to maintain the posture during upright standing. This is coordinated by the central nervous system with input from three systems: visual, vestibular, and somatosensory (or the proprioceptive system). Body sway is used as an indicator of postural stability. Various neurological and musculoskeletal diseases are related to impaired balance which results in an increased risk of falling due to deficits of

the proprioceptive system or muscle weakness. Falls due to impaired postural control present a serious health hazard to the elderly as well as to persons with balance disorders. Impaired balance diminishes a person's ability to perform activities of daily living.

Three test protocols are used in clinical Computerized Dynamic Posturography developed by NeuroCom for diagnosing the functional impairments underlying balance disorders: **The Sensory Organization Test** is intended to assess the patient's ability to make effective use of visual, vestibular, and somatosensory information and to appropriately suppress disruptive visual and/or somatosensory information under sensory conflict conditions. **The Motor Control Test** is intended to assess the patient's ability to reflexively recover from unexpected external provocations quickly and with appropriate movement patterns. **The Adaptation Test** is intended to assess the ability to modify reflexive motor reactions when the support surface is irregular or unstable. Most other Computerized Dynamic Posturography devices quantify postural stability using force plate technology. These devices measure the ground reaction force with transducers attached to a force plate to determine the center of pressure (COP). The upward projection of the COP is used as an estimate for the body center of mass (COM). Different low-pass filters are used on the COP time series to remove the high frequency content [1, 2], based on the assumption that postural sway is quasi-static. Another approach is to estimate COM with the second integral of horizontal acceleration, which is assumed to be proportional to the horizontal ground reaction force [3]. However, this method requires the estimation of initial conditions [4]. Winter et al estimate COM based

on their 14-segment COM model and measurements at 21 markers [5]. While this approach is good for research purposes, it is less practical for clinical use.

Another reason for obtaining COM is for evaluating the ankle postural stiffness [5, 6]. This evaluation requires the determination of moment produced at the ankle for maintaining posture. In most of the studies, only the moment equilibrium is considered whereas equilibrium in horizontal and vertical directions is ignored in the system equations.

This study is aimed at evaluation of ankle postural stiffness based on balance test data collected with a NeuroCom's EquiTest device [7]. (Disclaimer: None of the authors has a vested interest in the EquiTest or NeuroCom.) This system was developed to run on the early personal computers of the 1980s and consequently its calculations are as numerically simple as possible. The operating principle of this device is described in Appendix A. During our investigation, we noticed several shortcomings in the generation of COM by the device. One shortcoming is that this device uses the moving average of the COP as an estimate for COM. As mentioned earlier, this estimation is only good for quasi-static standing, while some of the test conditions should be treated as dynamic, because most individuals show considerable sway, particularly when the platform moves. Another problem is that the shear force, although measured, is not used in the estimation of COM. Thus, the computation performed by the device does not take rotation of the force plate into consideration, which, as we will show, produces an incorrect estimation of the COM.

Our work is intended to correct the above shortcomings by developing a mathematical model for quantifying COM directly from the measurement of ground reaction force, while taking into account of the rotation of the force platform. We also propose using the model to study the stiffness of the muscles around the ankle and their relation to the destabilizing force of gravity on the human body. The utility of our computational method to generate COM, ankle stiffness and other information is demonstrated with an example.

Method

As described in Appendix A, the generation of balance test results by an EquiTest device involves two steps. First, its data acquisition hardware produces ground reaction forces from 5 transducers, sampled at 100Hz. Then, its software processes the time series of force measurement to generate various reports for each trial of every test conditions (see Appendix A). The reported manufacturer's specification for the resolution of a force transducer, together with its amplifier and analog-to-digital conversion, is 0.87 N (0.195 lb). However, we believe that there are shortcomings in the calculations made by the device. Thus, our approach is to develop a new computational method based on the output of the data acquisition hardware (i.e., the quantization levels of the force transducers). The results are then used to evaluate COM, ankle moment, ankle muscle stiffness, and net moment around the ankle.

In order to better understand the calculations made by the device and the problem therein, we first review the dynamics of the "human inverted pendulum" for sway in the sagittal plane. Figure 1 shows the entire body excluding feet as an inverted pendulum rotating about the ankle joint A. M is the mass of body above the ankle, $F_{H,A}$ and F_V are horizontal and vertical forces acting at the ankle joint, τ is the resultant moment acting at the ankle joint by muscles and passive structures (ligaments, cartilage) around the ankle and θ is absolute sway angle with respect to a fixed vertical reference.

Figure 2 shows the feet together with the force plate. m is the total mass of the feet and the force plate; F_F and F_R are ground reaction forces perpendicular to the force plate, measured with front and rear transducers, respectively; F_H is the ground reaction force parallel to the force plate, measured with a transducer at the pin joint; d is the distance between the pin axis and the transducers that measure forces perpendicular to the force plate; e is the distance between the ankle joint and the top surface of the force plate; a (not shown) is the perpendicular distance between the line through the ankle and pin joints and the center of mass of the feet, θ_m is the sway angle of the center of mass relative to the line perpendicular to the force plate, and ϕ is the inclination angle of the force plate.

The calculation by the device does not take into account the inclination, ϕ, of the force platform and the shear force, F_H. In fact, the only use of shear force measurement by the device is to obtain the "strategy score", which is viewed as an indicator of the involvement of hip-sway (instead of ankle sway) in maintaining balance. We believe the

contribution by the inclination of the platform to shear force should not be ignored. Let us illustrate the effect of inclination on shear force with the sample data shown in Appendix A. For data point 1998, we have $F_F + F_R = 196 + 256 + 246 + 151 = 849$ (quantization levels) and $F_H = 68$ (quantization levels). The ratio $F_H / (F_F + F_R) = 0.080$ is the result of sway dynamics and the inclination of the force platform. The contribution from the inclination of the force platform may be seen from static situations. If the inclination of the force platform is ϕ, then $F_H / (F_F + F_R) = \tan\phi$ in static situations. It may be noted that if $\phi = 2°$, $\tan\phi = 0.035$ and if $\phi = 5°$, $\tan\phi = 0.087$. Comparing these values to the ratio of 0.080 in the above example, we see that the inclination of the force platform could contribute a significant portion of the total shear force.

To address the above problem, we first derived a complete set of dynamic equilibrium equations (B10-B12, in Appendix B) to establish the relationship between sway movement and the ground reaction forces. Parameters M, m, I (the body's moment of inertia about ankle joint), h, e and a in the equations are functions of a subject's height and weight and are calculated using anthropometric data taken from the literature [8-10].

The set of dynamic equations is then solved to obtain equation (B15), which is a nonlinear equation involving the absolute sway angle, θ. A simple solution (B19) is obtained by applying a small angle approximation in equation (B15). We have compared the solution obtained with and without the approximation and the approximation resulted in negligible change. The error generated by the small angle approximation is less than

0.13%, 0.51% and 1.2% for θ of 5°, 10° and 15°, respectively. The relative sway angle, if desired, can be obtained as $\theta_m = \frac{\theta}{k+1}$ (gain k is defined in Appendix A).

The EquiTest device we use does not produce information on the ankle moment. In our computation, we include the evaluation of the ankle moment (B20) so that we can study the relationship between the moment and angular sway at the ankle joint. We must use only the portion of the ankle moment responsible for the elastic deformation at the ankle joint in the determination of ankle postural stiffness. Our results suggest that there might be a linear relation between the ankle moment and the sway angle when the platform is fixed. This leads us to perform a linear regression of the ankle moment versus the sway angle, in the form of $\tau = k_p\theta + k_d\dot{\theta} + k_c$, to the test data obtained under such conditions with platform fixed. The term $k_p\theta$ is the elastic component of the ankle moment, $k_d\dot{\theta}$ is the viscous component of the ankle moment, and k_c represents the constant component. Results of this regression are discussed in the following sections. Since our results show little correlation between τ and $\dot{\theta}$, the relation $\tau = k_p\theta + k_d\dot{\theta} + k_c$ reduces to $\tau = k_p\theta + k_c$. This implies that the ankle moment due to the viscous component is negligible. The slope k_p of the linear regression is thus the ankle postural stiffness during quiet standing.

Postural sway is the result of interaction between the ankle moment and the destabilizing moment τ_g of gravity. The time history of the net moment might be able to reveal

additional information about the sway. For this reason, we also looked into the time history of the net moment $\tau_g - \tau$.

We implemented our new computational method with MATLAB. The input to our MATLAB program is the data files, which contain the quantization levels of force transducers, generated by an EquiTest device. This program is tested with data from four healthy adult subjects (two males and two females, ages between 29 and 70). One of the subjects is an author of this paper. Other subjects provided informed consent using forms approved by UMDNJ IRB. Each of the four sets of complete Sensory Organization Test data contains 18 20-second trials (3 trials for each of the 6 conditions, see Appendix A).

When the above method is used to process the clinical data, a detailed error analysis will be needed to determine the degree of precision in the computed sway angle and the ankle moment caused by the errors in force measurement. We did not perform error analysis in this study, because the numerical results presented in the following section is used only to illustrate the utility of our method.

Results

This study focuses on the method to be used for the evaluation of a subject, not on the specific results from any individuals or groups. Since similar results are obtained for all four subjects, only the results from one subject (one of the authors) are presented here to illustrate the utility of our method. This male subject's height and weight are $H = 1.67\ m$ and $W = 740.66\ N$, respectively. Other parameters for the subject are found as

$M = 73.235\ kg$, $m = 2.265\ kg$, $I = 85.02\ kg \cdot m^2$, $h = 0.933\ m$, $d = 0.107\ m$, $e = 0.065\ m$ and $a = 0.0315\ m$. Two of the 18 trials are presented here. One represents the test conditions where the platform is fixed ($k = 0$) and the other represents the test conditions where the platform is "sway referenced" and rotates the same angular amount that the COM moves ($k = 1$).

Figure 3 shows our computed COM ($y = h \cdot \theta$), labeled as "our result", and the device reported COM, labeled as "moving average", by the device for one trial with $k = 0$. Although both curves are based on the same measured forces, the computed curve differs quantitatively from the curve generated by the device because the computational methods are different. Since there is no inclination in the force platform and the shear force is negligible when the platform is fixed, the difference between the two curves is relatively small. The device estimates h as $0.5527H$ or $h = 0.923m$, which is slightly smaller than the value ($h = 0.933\ m$) we used. As a result, the reported COM curve is slightly lower (about 1%) than our computed COM. We can see clearly the smoothing effect of the moving average in the device reported COM.

Figure 4 shows our computed COM, both the absolute sway $y = h \cdot \theta$ (with respect to a fixed vertical reference) and the relative sway $y_m = h \cdot \theta_m$ (relative to the line perpendicular to the force plate), and the reported COM by the device for one trial with $k = 1$. The COM reported by the device is also relative to the line perpendicular to the force plate. Because of the rotation of the platform, this trial involves the inclination of the force platform and a noticeable shear force. The difference between our computed

COM and the reported COM can clearly be seen and is due to the inclusion of the shear force and the rotation of the platform in our analysis.

The time series of the computed ankle moment for the same two trials (platform fixed and moving) is plotted separately in Figures 5 and 6. Comparing the plots of COM in Figure 3 and ankle moment in Figure 5, the similarity of the two plots is apparent. This similarity is consistent among all other subjects when the platform is fixed. Comparing the plots of COM in Figure 4 and ankle moment in Figure 6, the plot of ankle moment is similar to the plot of the device reported COM, but has no similarity to the computed COM.

The result of the linear regression of the ankle moment versus the sway angle for the trial presented in Figure 3 is shown in Figure 7. The correlation between τ and θ is confirmed by coefficient $r_{\tau\theta} = 0.996$, but there is little correlation between τ and $\dot{\theta}$ since coefficient $r_{\tau\dot{\theta}} = -0.059$. The equation of the line is $\tau = 655.73 \cdot \theta + 0.978$ $(N \cdot m)$ with the coefficient of determination $R^2 = 0.993$. The ankle postural stiffness is therefore 655.73 $N \cdot m / rad$ for sway with the fixed platform. The goodness-of-fit of the linear regression can be estimated with the chi-square probability of the fit, which requires the knowledge of the measurement errors and their distribution [11]. A simple means we use to evaluate error in parameters (the slope and the intercept) obtained from linear regression is the standard deviation of the fit, which is approximately the average difference between each data point and the best-fit line. In the case of Figure 7, the standard deviation of the fit is 0.10 Nm. With the ankle moment between 40.4 to 46.1 Nm

during the trial, the standard deviation of the fit is less than 0.25%. We can also check the standard deviation of the slope and the standard deviation of the intercept, which are approximately the difference in slope and intercept, respectively, between the best-fit line and a limiting reasonable fit line. The standard deviation for the slope is 1.23 Nm/rad, which is about 0.2% of the slope. The standard deviation for the intercept is 0.08 Nm, which is about 8% of the intercept.

The moment produced by the gravitational force can be represented by another line $\tau_g = Mgh \cdot \theta = 670.22 \cdot \theta$ ($N \cdot m$). The time history of the net moment $\tau_g - \tau$ for the same trial is presented in Figure 8. It shows that the net moment is very close to zero during the entire trial. However, the bit error in the force measurement makes the actual values of the net moment meaningless. The discrete nature of the force measurement due to analog-to-digital conversion was masked in the plotted ankle moment, but is revealed in this plot. If the true nature of the net moment is to be revealed, a much improved resolution in force measurement is needed.

Table 1 shows one example of regression of the ankle moment vs. the sway angle for each of the four subjects, all under the condition that the platform is fixed and eyes open. All the regression results show high values of the coefficient of determination. We can conclude from this that the ankle moment varies linearly with sway angle, for all 4 subjects when the platform is fixed.

Discussion

The computational method we developed has two important features. One is that the solution is obtained without either filtering or numerical integration. The other is the inclusion of shear force and rotation of the platform. This computational method can also be applied to the situations where the platform is fixed and inclined (i.e., ϕ is a nonzero constant). In our computations, we do not ignore parameters F_H, e, m, and a as done in the previous studies by others, such as Morasso and Schieppati [~~11~~12] and Winter et al [5].

Several factors can affect the results obtained with our computational methods. One factor is the accuracy of ground reaction force measurement. For the NeuroCom device we use, the resolution of the force measurement is about 0.87 N for each transducer. This limited resolution leads to significant error when the value of quantization levels is small. For example, the back and forth sway motion indicates the existence of angular acceleration. However, meaningful calculation of angular acceleration cannot be achieved with the current resolution of force transducers when the platform is fixed. Because the sway angle is small, the magnitudes of the two terms $[(M+m)g-(F_F+F_R)]\sin\theta$ and $F_H\cos\theta$ in equation (B13 with $\phi=0$) are really in ranges comparable to each other. When the platform is fixed, $F_H=0$ is reported and $B=0$ in equations (B15-B17). The values of $\sigma=\pm1$ in (B18) produce two supplementary solutions of angle θ. Another factor is the potential rotation at other joints. Our computation is based on the simple inverted pendulum model of postural sway, which assumes that the rotation happens only at ankle joints. While we observed

no obvious hip movement during our experiments, this assumption requires further validation through experiments with multi-link models. This validation, which is the subject of our future research, will help to better define the operating range of the device.

The moving average displacement in Figure 4 is closer in shape to the ankle moment in figure 6 than the displacement computed with our method. We might expect this result if we approximate the moving average displacement by ignoring the horizontal force component in equation (B19) and compare it with the ankle moment in equation (B20). The need to account for shear forces in the computation becomes evident when we take a close look at equations (B19) and (B20). Let us again use the data point 1998, from one trial of our example subject, in Appendix A. We have $F_F - F_R = 196 + 151 - 256 - 246 = -155$ (quantization levels) and $F_H = 68$ (quantization levels). The two equations produce $\theta = 0.302\,(rad) = 17.3°$ and $\tau = -11.3\,(Nm)$. If F_H is ignored, we obtain $\theta = -0.069\,(rad) = -4.0°$ and $\tau = -15.1\,(Nm)$ instead. Thus ignoring F_H leads to a significant difference in the results. The different shapes of the ankle moment and the displacement computed with our method suggest that a linear relationship between them no longer holds when the plate is rotating. How the moment of the shear force and the inertia moment of the body interact at the ankle joints requires further investigation, if the simple inverted pendulum model is not applicable. Joint position data for multiple joints obtained from a system other than the balance test will be needed to develop such a model.

We conclude from Figures 5 and 6 that very different ankle moments are required, depending on whether platform is fixed or moving. The maximum ankle moment generated with the moving platform is almost double that generated with the fixed platform in these two trials. It is also observed that negative ankle moment is generated with the moving platform in contrast to the fixed platform where there is no negative moment.

Test conditions with the platform fixed represent the quiet standing studied by Winter et al [5, 6]. Our results for ankle moment show that the ankle stiffness closely resembles an ideal spring, which is in agreement with [6]. In our linear regression, both ankle moment and sway angle are calculated from the same measured ground reaction forces. Values of the ankle stiffness and the coefficient of determination R^2 vary in different trials and conditions. The coefficient of determination reported in [6] is $R^2 = 0.954$, where ankle moment and sway angle were obtained from separate measurements. Both the "moving average" and "our result" would produce similar fit from the linear regression. What is gained with our method is that the details of the sway and the ankle moment are not being smoothed out. These details could become useful in finding better outcome measures of postural balance.

Although the slope $655.73\ N \cdot m / rad$ of the regression line in Figure 7 differs from that ($670.22\ N \cdot m / rad$) of the moment τ_g produced by gravity, the net moment $\tau_g - \tau$ is less than 1% of the ankle moment, as shown in Figure 8. Intersection of these two moment lines occurs at $\theta = 0.0675\,(rad) = 3.87°$ for the trial in Figures 3. The

corresponding COM displacement is $0.063\,m$, which represents a critical point of the stability. Thus the sway motion should not be allowed to deviate beyond this point too much to ensure stability.

Our computation of the COM for sway-referenced motion (platform rotating) produces significantly different results when compared with those reported by the device. We believe the computation by the machine is incorrect for these conditions since it ignores the effect of the shear force as well as the mass and the rotation of the force plate. As long as the simple inverted pendulum model is still appropriate for the sway-referenced motion, our method will produce the correct results. In our formulation, we assume that the rotation of the force plate is precisely servo-controlled to follow the sway of the subject. In reality, there should be a time delay of one sampling period (10 ms in this case) in obtaining the sway angle for position control. For sway-referenced motion (platform rotating), we simply showed the result of the calculation under the assumption that the model is applicable. The relationship between the ankle moment and sway angle during sway-referenced motion requires further study.

Conclusion

The new computational method corrects shortcomings in an existing method for evaluating postural stability by including inclination of the platform and the shear component of the ground reaction force in the mathematical model. Based on this model, the solution of postural sway is obtained without either filtering or numerical integration.

In this method, ankle moment and ankle postural stiffness for quiet standing are also evaluated. Factors that could affect the application of this method are discussed.

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Appendix A

The EquiTest device has been available commercially since 1986. This device performs a battery of tests: Sensory Organization Test, Motor Control Test and Adaptation Test. It consists of a movable dual platform capable of anterior-posterior translation and rotation about the ankle joints in the sagittal plane. The dual platform is surrounded by a visual scene that can also rotate about the ankle joints. The EquiTest device quantifies the ground reaction force (the force exerted by the platform to each of the feet) using five force transducers. The two force plates are connected by a pin joint and supported by four force transducers mounted symmetrically on a supporting plate. These four transducers measure forces perpendicular to the force plate. A fifth transducer is mounted to the supporting plate directly beneath the pin joint for measuring shear force. The force transducers are sampled at 100Hz. The sampled data is recorded as integer quantization levels in the following form (data from one of our subjects):

Number of Data Sample: 2000

DP	LF	RR	SH	LR	RF
1	319	108	0	110	314
2	319	106	0	110	314
3	318	106	0	110	314

…	…				
1998	196	256	68	246	151
1999	197	256	68	247	151
2000	197	258	68	247	151

where DP means data point, LF and RF represent quantization levels from transducers mounted at left and right front, LR and RR represent quantization levels from transducers mounted at left and right rear, and SH represents the quantization level from the transducer at the pin that measures shear force. The resolution of the transducers is about 0.87 N between quantization levels. F_F and F_R in Figure 2 are, respectively, the force values converted from LF+RF and LR+RR. F_H is the force value converted from SH. The device determines the center of pressure (COP) in the sagittal plane from the formula:

$$COP = \frac{(LF + RF) - (LR + RR)}{LF + RF + LR + RR} \times 4.20 \quad (inches) \tag{A1}$$

The center of mass (COM) at any particular moment is estimated with a one-sided moving average filter as

$$COM(i) = \frac{1}{M} \sum_{j=1}^{M} COP(i - j) \tag{A2}$$

In this equation, i is the index of the COM data series, $COP(i - j)$ $(j = 1, \cdots, M)$ are the COP values obtained with equation (A1) from the past M measurements of the transducers and $M = 14$ is the number of points used in the moving average.

The device then computes sway angle as

$$\theta = \arcsin\left(\frac{COM}{0.5527 * H}\right) - 2.3° \tag{A3}$$

where H is the height of the subject and 2.3˚ is the so called "forward lean" of the angle of the COM.

The sensory organization test (SOT) is comprised of 6 test conditions used to assess the vestibular, proprioceptive and visual aspects of balance. Participants stand on a computer-controlled platform within and facing a semi-circular visual surround. For sway-referenced conditions, the movement of the surround or platform is referenced to the individual's sway. For example, in the sway-referenced surround condition, if the person sways forward, the surround also sways forward, reducing the degree to which the individual can use visual cues to perceive they are no longer vertical. Participants are asked to stand quietly and steadily for 3 trials each of the following 6 conditions: (1) eyes open, surround and platform stable, (2) eyes closed, surround and platform stable, (3) eyes open, sway-referenced surround, (4) eyes open, sway-referenced platform, (5) eyes closed, sway-referenced platform, and (6) eyes open, sway-referenced surround and platform. The rotation of the base that supports the force transducers and the force plates is controlled by an actuator. This rotation is proportional to the sway angle θ. An operator can set the proportional constant *k*, which is referred to as "Gain", in the range of -1.0 ~ 2.0. The value of *k* is 0 when the platform is fixed.

The SOT equilibrium score (ES) is the angular difference between the calculated maximum anteroposterior displacement of the center of gravity and the subject's theoretical maximum sway of 12.5˚ expressed as a percentage for each trial. Equilibrium

scores closer to 100 indicate better balance and those closer to zero worse. A fall counts as a zero for the trial.

$$ES = \left(1 - \frac{\theta_{\max} - \theta_{\min}}{12.5°}\right) \cdot 100\% \tag{A4}$$

The overall SOT equilibrium score is calculated by first averaging the 3 scores for each of conditions 1 and 2, and then adding these 2 averaged scores to the scores for each trial of Conditions 3-6 and dividing the total by 14. This weights the more difficult conditions (Conditions 3-6) more than the easy conditions (1 and 2) in the overall score.

The SOT Strategy Score (SS) is based on the peak-to-peak of the measured horizontal (shear) force normalized to a maximum of 25 pounds.

$$SS = \left(1 - \frac{sh_{\max} - sh_{\min}}{25}\right) \cdot 100\% \tag{A5}$$

In this equation, *sh* is the shear force in pound converted from the measured SH. The Strategy Score is used as an indicator of the involvement of hip-sway in maintaining balance.

Appendix B

In this appendix, we present a derivation of the formulas for the proposed computational method. Nomenclature used here is introduced in connection with Figures 1 and 2.

The equation of motion for the body in Figure 1 can be written as

$$F_{H,A} = M\ \ddot{x}_c = M\ h(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta) \tag{B1}$$

$$F_V = M(g + \ddot{y}_c) = Mg - Mh(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \quad \text{(B2)}$$

$$-\tau + F_V h\sin\theta - F_{H,A} h\cos\theta = I_c\ddot{\theta} \quad \text{(B3)}$$

where $\ddot{x}_c$ and $\ddot{y}_c$ are respectively the horizontal and vertical acceleration at COM. I_c is the moment of inertia about the COM and h is distance between COM and the ankle joint. By substituting equations (B1) and (B2) into (B3) and simplifying, we have

$$-\tau + Mgh\sin\theta = (I_c + Mh^2)\ddot{\theta} = I\ddot{\theta} \quad \text{(B4)}$$

where I is the moment of inertia about ankle joint.

The equation of motion for the feet in Figure 2 can be written as

$$F_{H,A} = F_H\cos\phi + (F_F + F_R)\sin\phi \quad \text{(B5)}$$

$$F_V = (F_F + F_R)\cos\phi - F_H\sin\phi - mg \quad \text{(B6)}$$

$$\tau - (F_F - F_R)d - F_H e + mga\cos\phi = 0 \quad \text{(B7)}$$

The sway angle computed by the device is θ_m, which is based on the reading of the vertical force transducers. Its reference is the line perpendicular to force plate. As mentioned in Appendix A, the inclination angle ϕ is related to the computed sway angle θ_m with a gain factor (-1.0 ~ 2.0) as

$$\phi = k \cdot \theta_m \quad \text{(B8)}$$

The absolute sway angle θ can be expressed as

$$\theta = \phi + \theta_m \quad \text{(B9)}$$

After eliminating all the internal forces and moments $F_{H,A}$, F_V, and τ from equations (B1-B7), we have the following three equations for the postural system:

$$Mh(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta) = F_H \cos\phi + (F_F + F_R)\sin\phi \tag{B10}$$

$$Mh(\ddot{\theta}\sin\theta + \dot{\theta}^2 \cos\theta) = (M+m)g - (F_F + F_R)\cos\phi + F_H \sin\phi \tag{B11}$$

$$I\ddot{\theta} = Mgh\sin\theta - (F_F - F_R)d - F_H e + mga\cos\phi \tag{B12}$$

Since the ground reaction forces F_F, F_R and F_H are measured with the force transducers and the base rotation ϕ is under specified machine control, we can obtain the unknown state θ, $\dot{\theta}$, and $\ddot{\theta}$ of the sway at each sampling point by solving equations (B10-B12). From equations (B10) and (B11), we have

$$\ddot{\theta} = \frac{(M+m)g\sin\theta + (F_F + F_R)\sin(\phi-\theta) + F_H \cos(\phi-\theta)}{Mh} \tag{B13}$$

$$\dot{\theta}^2 = \frac{(M+m)g\cos\theta - (F_F + F_R)\cos(\phi-\theta) + F_H \sin(\phi-\theta)}{Mh} \tag{B14}$$

Note that the solution only yields the magnitude $|\dot{\theta}|$, not the direction, of the angular velocity $\dot{\theta}$. Therefore, this direction information must be derived from the time history of angle θ. Substituting (B13) into (B12), and using $\phi = \frac{k}{k+1}\theta$, we have

$$\begin{aligned}&\left[I(M+m) - M^2h^2\right]g\sin\theta - I(F_F + F_R)\sin\frac{\theta}{k+1} + F_H I\cos\frac{\theta}{k+1}\\ &- Mhmga\cos\frac{k\theta}{k+1} + Mh\left[(F_F - F_R)d + F_H e\right] = 0\end{aligned} \tag{B15}$$

For the test conditions where the platform is fixed, we have $k = 0$ and equation (B15) becomes

$$\left[I(M+m)g - I(F_F + F_R) - M^2 g h^2\right]\sin\theta + F_H I \cos\theta + Mh\left[(F_F - F_R)d + F_H e - mga\right] = 0 \tag{B16}$$

which can be expressed as

$$A\sin\theta + B\cos\theta + C = 0 \tag{B17}$$

where $A = I \cdot \left[(M+m)g - (F_F + F_R)\right] - M^2 h^2 g$, $B = F_H I$, and $C = Mh\left[(F_F - F_R)d + F_H e - mga\right]$.

The solution to equation (B17) can be found, by making substitutions $t = \tan\frac{\theta}{2}$, $\sin\theta = \frac{2t}{1+t^2}$, and $\cos\theta = \frac{1-t^2}{1+t^2}$, as

$$t = \tan\left(\frac{\theta}{2}\right) = \frac{-A + \sigma\sqrt{A^2 + B^2 - C^2}}{C - B} \tag{B18}$$

where $\sigma = \pm 1$ identifying the solution mode. The location of the COM can be obtained as $y = h\sin\theta = h\frac{2t}{1+t^2}$.

For the test conditions where the platform is rotating, we have $k \neq 0$ and the solution to equation (B15) involves more complex computation. A small angle approximation may be employed to simplify the computation. With this approximation, equation (B15), reduces to

$$\theta = \frac{Mh[(F_F - F_R)d + F_H e - mga] + I \cdot F_H}{M^2 gh^2 - I\left[(M+m)g - \frac{F_F + F_R}{k+1}\right]} \tag{B19}$$

The ankle moment can be evaluated with equation (B7) for all test conditions as

$$\tau = (F_F - F_R)d + F_H e - mga\cos\frac{k\theta}{k+1} \tag{B20}$$