Analysis of Stress and Pressure in the Human Alveolar Wall Before Bursting

H. R. Chaudry\(^{(1,2)}\), B. Bukiet\(^{(3)}\), and S. Kirshblum\(^{(4)}\)

\(^{(1)}\) War-Related Illness and Injury Study Center  
VA Medical Center, East Orange, NJ

\(^{(2)}\) Department of Biomedical Engineering  
New Jersey Institute of Technology, Newark, NJ 07102

\(^{(3)}\) Department of Mathematical Sciences and  
Center for Applied Mathematics and Statistics  
New Jersey Institute of Technology, Newark, NJ 07102

\(^{(4)}\) Spinal Cord Injury Program  
Kessler Institute for Rehabilitation  
West Orange, NJ

CAMS Report 0304-12, Fall 2003

Center for Applied Mathematics and Statistics

NJIT
ANALYSIS OF STRESS AND PRESSURE IN THE HUMAN ALVEOLAR WALL BEFORE BURSTING

H.R. CHAUDHRY  
VA Medical Center  
East Orange, NJ 07018  
and  
Department of Biomedical Engineering  
New Jersey Institute of Technology  
Newark, NJ 07102  
chaudhry@adm.njit.edu

B. BUKIET  
Department of Mathematical Sciences  
Center for Applied Mathematics and Statistics  
New Jersey Institute of Technology  
Newark, NJ 07102  
http://m.njit.edu/~bukiet/  
bukiet@m.njit.edu

S. KIRSHBLUM  
Spinal Cord Injury Program  
Kessler Institute for Rehabilitation  
West Orange, NJ 07052  
skirshblum@KESSLER-REHAB.COM

A strain energy function for the human alveolar wall in vivo is developed based on its length-tension properties. Using large deformation theory, this function is employed to determine the relationship between circumferential stress and stretch ratio as well as between alveolar pressure and stretch ratio. We find that both the circumferential stress and the pressure in the alveolar wall rise very rapidly if the wall is stretched even slightly more than that which occurs at the pressures attained during mechanical in-exsufflation (MI-E). MI-E involves producing a high inspiratory air pressure and then quickly switching to a large negative air pressure in order to remove mucus from the airways.

Keywords: Human Alveoli, Bursting, Circumferential Stress

Running Head: Bursting Pressure and Stress of Human Alveoli

1. Introduction

The basic structural unit of the lung is the alveolus, which is capable of large deformations when subjected to internal pressure (Fung, et al., 1978). The alveolar wall consists of collagen and elastin fibers. The mechanical characteristics of the alveolar wall (intraalveolar septa) play an important role in the overall performance of the whole lung. (Gafen, et al., 1999). Each alveolus is responsible for exchange of oxygen and carbon dioxide between blood and air during respiration. The diameters of the alveoli vary from 100 µm to 300 µm and the wall thickness from 1 µm to 5 µm (Fung, et al., 1978).

Stresses in the alveolar wall, alveolar pressure and the stretch ratio of the wall (a measure of its deformation) are all closely related. The alveolar pressure is influenced by gravity, posture and chest wall and diaphragm motion during respiration. Redistribution of stresses in the alveolar wall is one of the major causes of pulmonary diseases, such as emphysema (permanent enlargement of air spaces distal to the terminal bronchioles), spontaneous collapse and bursting of the alveoli and pulmonary tuberculosis (Fung, et al., 1978; Negai et al., 1991; Denny and Schroter, 1995).

The mechanical characteristics of the whole lung have been studied by various models (Dale, et al., 1980; Fung, 1988; Denny and Schroter, 1995). However, these efforts were not focused on studying the internal stress distribution at the alveolar wall. The internal stress distribution at the alveolar wall for normal and emphysematic lungs have been investigated by Gafen, et al., (1999) by developing a two dimensional in vitro model of a typical alveolar sac structure. Our present in vivo model is, however, different from that of Gafen, et al., 1999 in the sense that we model the alveolus as a three-dimensional thin spherical shell.
The main objective of the present work is to understand the relationship among pressure, stress and stretch ratio at stretch ratios near the point where the alveoli burst. Patients with restrictive respiratory disease, including those with spinal cord injuries, cystic fibrosis and motor neuron diseases including amyotrophic lateral sclerosis (ALS) and poliomyelitis, experience a build up of mucus in the lungs and have difficulty mobilizing their secretions (cough the mucus out). These patients benefit from a non-invasive technique called mechanical in-exsufflation (MI-E). This technique involves producing a high inspiratory air pressure and then quickly switching to a large negative air pressure in order to remove mucus from the airways. This procedure has been shown to be more effective and better tolerated than the standard of suctioning commonly used for patients with a tracheostomy tube (Sancho 2003, Garstang 2000, Bach 1994). It is important to investigate the relationship among circumferential stress, alveolar pressure and stretch ratio to explore the physiological safety of MI-E.

In this paper, the length-tension properties of the human alveolar wall (Sugihara, et al. 1971) are used to develop a strain-energy function based on a large deformation theory. This function is then employed to determine the relationship between alveolar pressure and stretch ratio as well as between circumferential stress and stretch ratio in a single alveolus modeling it as a thin spherical shell. Interactions among alveoli and the influence of surface tension and surfactants on the behavior of alveoli are not considered.

2. Method

Since alveoli experience large deformations when subjected to internal pressure, we employ a large deformation theory (Green and Zerna, 1968). The uni-axial data for the length-tension properties of the alveolar wall of a healthy 22 year old woman (Sugihara, et al., 1971) is used to find the elastic parameters involved in the strain-energy function.

We assume the alveolar wall to behave like a neo-Hookean solid (a rubber-like material). Under this assumption, the strain energy, \( W \) becomes a function of strain invariant \( I_1 \) only. Thus, we take the strain-energy function to be

\[
W = \frac{C_1 (I_1 - 3)}{a - I_1}
\]  

(1)

where \( I_1 = \frac{\lambda^2 + \frac{2}{\lambda}}{a} \) is the strain invariant in the uni-axial case (Green and Zerna, 1968, p. 80), \( \lambda \) is the extension ratio, and \( C_1 \) and \( a \) are elastic parameters that need to be computed.

The uni-axial stress, \( \sigma_{11} \), is given by (Green and Zerna, 1968, p. 80)

\[
\sigma_{11} = (\lambda^2 - \frac{1}{\lambda})(2 \frac{\partial W}{\partial I_1})
\]  

(2)

Using \( \sigma_{11} = \frac{F}{A} \), and \( AL = A_0 L_0 \) (assuming incompressibility of the alveolar wall) where \( F \) is the applied force, \( A_0 \) and \( A \) are the areas of cross-section of the specimen in the undeformed and deformed states, respectively, \( L_0 \) and \( L \) are the lengths in the undeformed and deformed states, respectively and \( L = \lambda L_0 \), we evaluate the force, \( F_{model} \) using Eqs. (1) and (2).

\[
F_{model} = 2A_0 (\lambda - \frac{1}{\lambda^2}) \frac{\partial W}{\partial I_1} = \frac{2A_0 (\lambda - \frac{1}{\lambda^2})}{(a - I_1)^2} [C_1 (a - 3)]
\]  

(3)
where \( A_0 \) is \( 9.9 \times 10^{-6} \text{ cm}^2 \) from experiments (Sugihara, 1971). Sugihara found the force for an \textit{in vitro} sample of alveolar tissue experimentally, \( F_{\text{exp}} \). We compute \( a \) and \( C_1 \) by minimizing the square of the differences of the model and experimental force values (numerical least squares method). We find the best values to be \( a=5.691 \) and \( C_1=40.178 \text{ mmHg} \). A plot of the \textit{in vitro} force vs. stretch ratio is presented in Figure 1. Also presented in Fig. 1 is a plot of force vs. stretch ratio \textit{in vivo}, computed as described in the 3-D model section (section 3) below.

\[ \frac{\partial}{\partial x} - \frac{1}{\lambda} \left( \frac{1}{\lambda^2} - \frac{1}{\lambda^7} \right) \frac{\partial W}{\partial I_1} \]

\( \lambda \) is the ratio of the deformed radius to the undeformed radius. \( \lambda \) is also the cube root of the ratio of the deformed volume and the undeformed volume. Using Eqs. (1) and (4), we find

3. 3-D \textit{in vivo} model

Assuming the alveolus can be modeled as a thin spherical shell, the transmural pressure is given by (see for details Green and Zerna, p. 107, 1968)

\[ P_i - P_o = \frac{4h}{r} \left( \frac{1}{\lambda} - \frac{1}{\lambda^7} \right) \frac{\partial W}{\partial I_1} \]  

where \( P_i \) is the internal pressure and \( P_o \) is the external pressure, \( h \) and \( r \) are the thickness and radius of the alveolus in the undeformed state, respectively. For the spherical shell, the strain invariant, \( I_1 \), is given by

\[ I_1 = \frac{1}{\lambda^4} + 2\lambda^2 \]

(Figure 1. Experimental \textit{in vitro} (solid curve) force vs. stretch ratio plot based on Sugihara, et al., (1971) and \textit{in vivo} (dashed curve) force vs. stretch ratio based on physiological data (Levitsky, 1995), (Costanzo, 1998) and our 3-D \textit{in vivo} thin spherical shell model.)
We are interested in understanding the circumferential stress and pressure experienced by the alveoli in vivo before bursting. In order to do this, we find the total volume the alveoli can withstand based upon the data provided in Costanzo (1998, p. 166-7). The total lung volume is approximately 5.9 liters where about 150 ml is anatomical dead space. We assume here that the transmural pressure at this volume is 40 mmHg (Levitsky, 1995). The volume at atmospheric pressure is about 2.4 liters (including the anatomical dead space). This gives a ratio of total lung volume to atmospheric volume of 5.75/2.25 = 2.56. So, the stretch ratio is the cube root of 2.56, i.e., 1.37. Next, we computed the appropriate (linear) scaling of the in vitro data to make it relevant for the in vivo state. To do this, we recomputed the strain energy function parameters, $a$ and $C_1$, (for the in vivo state) for various (linear) scalings and found that scaling such that the pressure is computed as 40 mmHg when the stretch ratio is 1.37. We find, using Eqs. (5) and (6), that $a = 4.3$ and $C_1 = 27.09$ mmHg. The force vs. stretch ratio plot for both the in vitro and in vivo cases are presented in Fig. 1.

Using the method developed for circumferential stress, $\sigma_{\theta \theta}$, in Green and Zerna, (p. 105, 1968) we find

$$\sigma_{\theta \theta} = 2(\lambda^2 - \frac{1}{\lambda^2}) \left[ \frac{(a-3)C_1}{(a-I_1)^2} \right] - P_i$$

$$= 2(\lambda^2 - \frac{1}{\lambda^2}) \left[ \frac{(a-3)C_1}{(a-I_1)^2} \right] - \frac{4h}{r} \left[ \frac{\frac{1}{\lambda} - \frac{1}{\lambda^2}}{(a-I_1)^2} \right] = P_o$$

(7)

4. Results

Since the radius and thickness of alveoli are in the range 50 $\mu$m-150 $\mu$m and 1 $\mu$m – 5 $\mu$m, respectively, we use radius of 100 $\mu$m and thickness of 3.5 $\mu$m for our computations. Using $a=4.3$ and $C_1=27.09$, mmHg in Eqs. (6) and (7), we plot the graphs of transmural pressure, $P_i - P_o$, vs. stretch ratio and circumferential stress, $\sigma_{\theta \theta}$, vs. stretch ratio in Figures 2 and 3, respectively.
Fig. 2. Plot of pressure vs. stretch ratio \textit{in vivo} for a human alveolus based on the 3-D thin spherical shell model.
5. Conclusions and Discussion

We can see from Figures 2 and 3 that the transmural pressure and circumferential stress increases rapidly with stretch ratio. Eqs. (5) and (6), imply that pressure becomes infinite when $a = I_1$, i.e., $\lambda = 1.43$. Similarly, the circumferential stress becomes infinite at the same value of $\lambda$ based on Eqs. (5) and (7). Mathematically, the model breaks down at this point and physically bursting must occur before the stretch ratio gets this high.

We note that since transmural pressure of 40 mmHg is safe (Levitsky), an alveolar pressure of 50 mmHg is safe. This is because the intrapleural pressure, $P_o$, is 10 mmHg when the transmural pressure is 40 mmHg (Levitsky, 1995).

From the plot in Fig. 1, we observe that the in vivo alveoli are stiffer than the in vitro alveoli. This seems counterintuitive and might result from the fact that when the lung is inflated, the 300 million alveoli push on one another and cannot expand freely. We did not take into account interactions of the alveoli with one another in our model. In addition, the surface tension experienced by the walls of the alveoli was not taken into account in our model. With respect to the use of the mechanical in-exsufflator, we recommend based upon the above computations the use of a maximum inspiratory pressure of no more than 50 mmHg.

Acknowledgments

We thank Dr. J. R. Bach of the University of Medicine and Dentistry of New Jersey for useful discussions.

References