A shallow water ocean acoustics inverse problem

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CAMS Report 0203-13, Spring 2003

Center for Applied Mathematics and Statistics

NJIT

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March 7, 2003

Abstract
A shallow water ocean acoustics experiment is described from which it is possible to recover the scattering data necessary to recover the sound speed in the ocean bottom.

For this shallow water ocean acoustics inverse problem the following assumptions are made:
• The acoustic sound speed in the ocean and in the bottom depend only on the depth coordinate z, $c = c(z)$, for $0 \leq z < \infty$.
• The density is a constant for all z.
• The ocean occupies the region $0 \leq z \leq L$ where L is known, and the sound speed in the ocean is known. In an earlier paper[1], the sound speed in the ocean was also assumed unknown.
• The ocean bottom occupies the region $L < z < \infty$ and the sound speed there is unknown and is to be determined.
• The sound speed approaches a constant c_1 sufficiently rapidly as z approaches infinity and the constant c_1 is known.
The object is to determine $c(z)$ for $z > L$ from measurements in the ocean $0 \leq z \leq L$.
The experiment to determine the data to recover $c(z)$ is described below.
• At a depth z_0 in the ocean an isotropic, low-frequency (ω is its angular frequency) source projects acoustic energy. The frequency is chosen to be low enough so that no eigenmodes are excited. This means that only the continuous spectrum is excited.
• The pressure field is measured at the same depth for all range points that is for $0 \leq r < \infty$. The Hankel transform of the pressure field is calculated .
The Hankel transform of the pressure field satisfies (the transform variable is denoted by μ).
(Eq 1) $\frac{d^2 g(z,z_0,k}{d^2 z} + (k^2 - q(z))g(z, z_0, k) = \delta(z - z_0)$ for $0 \leq z < \infty$ and $0 < z_0 < L$, where
(Eq 2) $g(0, z_0, k) = 0$.
and
(Eq 3) $k^2 = (\omega/c_1)^2 - \mu^2$
and
(Eq 4) $q(z) = \omega^2(1/c_1^2 - 1/c^2(z))$
Let $S(z, k)$ and $U(z, k)$ both satisfy the homogeneous form of Eq 1 with
(Eq 5) $U(z, k) \sim e^{ikz}$ as $z \to \infty$
and
(Eq 6) $S(0, k) = 0$ and $\frac{dS(0,k)}{dz} = 1$.
The measured data needed to recover $q(z)$ for $L < z < \infty$ is [2],[3]
(Eg 7) $S_c(k) = U(0, -k)/U(0, k)$ for $0 \leq k < \infty$.
For k real $U(0, -k) = \overline{U}(0, k)$ and therefore the modulus of $S_c(k)$ is unity.
The green's function is given by
(Eq 8) $g(z, z_0, k) = -S(z, k)U(z_0, k)/U(0, k)$ for $z < z_0$
$= -S(z_0, k)U(z, k)/U(0, k)$ for $z_0 < z$
and since there are no eigenvalues $U(0, k) \neq 0$.
The measurement at z_0 yields $g(z_0, z_0, k)$ and will be denoted by g_M . In addition, since the sound speed is known in the ocean $S(z_0, k)$ can be calculated.

From Eq 8

(Eq 9) $U(z_0, k) = -g_M U(0, k)/S(z_0, k)$

For real k

(Eq 10) $S(z_0, k) = \Im(\overline{U}(0, k)U(z_0, k))/k$

hence

(Eq 11) $\mid U(0, k) \mid^2 = kS^2(z_0, k)/\Im\overline{g}_M$

But

(Eq 12) $U(0, k) = \mid U(0, k) \mid \exp i\Theta(k)$.

However, $\Theta(k)$ and $\ln(\mid U(0, k) \mid)$ are the real and imaginary parts of conjugate harmonic functions and hence Θ is given by the Cauchy principal value integral

(Eq 13) $\Theta(k) = (-2k/\pi) \int_{k'=0}^{\infty} \ln \mid U(0, k) \mid /((k')^2 - k^2)dk'$

and

(Eq 14) $S_c(k) = \exp(-2i\Theta(k))$.

References

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