Math 430 and Math 635: Analytical and Computational Neuroscience
Fall 2001, Prof. V. Booth
Take-Home Midterm Exam, Due Wednesday, Oct 24, 2001

1. The accompanying data file (see course website for link) contains measurements of the potassium current I_k in the squid axon during voltage clamp experiments that were conducted when the Na^+ current was blocked with tetrodotoxin (TTX). In the experiments, the voltage was initially at -80 mV and then was stepped up at time $t = 10$ msec to 4 different levels $V_c = -40$, -20, 0 and 40 mV. Voltage was held at V_c until $t = 150$ msec. Assume that I_k is accurately modeled by $I_k = g_k n^4 (V - E_k)$ where $g_k = 45$ mS/cm^2 and $E_k = -80$ mV but the gating variable n has different dynamics than in the Hodgkin-Huxley model.

   (a) Estimate values of the steady state activation function $n_\infty(V)$ for this axon at each voltage clamp V_c level.

   (b) Estimate values of the time constant function $\tau_n(V)$ at each voltage clamp V_c level. (Hint: From the solution of the differential equation for n, assuming $n = 0$ at $t = 0$, determine a relation for the value of n at time $t = \tau_n(V_c)$ and the steady state value $n_\infty(V_c)$.)

2. In numerical simulations of the Hodgkin-Huxley model using XPP or otherwise, investigate the voltage threshold for action potential generation. Apply long-lasting applied current steps to the model with amplitudes just smaller than needed to get repetitive action potential firing. Determine the highest steady state voltage level that can be reached before action potentials repetitively fire. This voltage level gives an estimate of the voltage threshold for action potential generation.

   In the function for $\alpha_m(V)$, the leading constant is -0.5. Change this constant to -0.1. Give **very** detailed descriptive answers to the following questions and support your conclusions with plots, numerical simulations, analytical computations, etc as needed. What is the effect of this change:

   (a) on $m_\infty(V)$?

   (b) on $\tau_m(V)$?

   (c) on the dynamics of m?

   (d) on I_{Na}?

   (e) on the voltage threshold for action potential generation?

   (f) Predict what would be the effect of only slightly increasing the constant. Support your conclusions with simulation results.