

**Math 430 and Math 635: Analytical and Computational
Neuroscience
Fall 2001, Prof. V. Booth
Suggestions for Final Projects**

1. For the Fitzhugh-Nagumo model

$$\frac{dV}{dt} = V(V - a)(1 - V) - w + I, \quad \frac{dw}{dt} = \epsilon(V - \gamma w), \quad (1)$$

where $0 < a < 0.5$, $0 < \gamma, \epsilon < 1$, do a linear stability analysis of the equilibrium solution to determine ranges of I where the solution is stable and unstable. Analytically determine all bifurcation points. Using XPP, construct a bifurcation diagram plotting V vs. I of stable and unstable solutions, steady and periodic. Provide phase plane plots of the different types of solutions found.

2. In the Fitzhugh-Nagumo model with the w equation given as above, there is no region of bistability around the Hopf bifurcation point - the Hopf bifurcation is supercritical. Determine how the w equation should be changed to make the Hopf bifurcation subcritical displaying a region of bistability. Keep the w nullcline linear but change its slope and intercept. Construct a bifurcation diagram and give an explanation in the phase plane on the differences between the two versions of the model.
3. Reduce the Hodgkin-Huxley model to a two-dimensional system by making the following assumptions:

$$m = m_{\infty}(V), \quad n = 0.82 - h. \quad (2)$$

Using XPP, determine ranges of I_{app} where the equilibrium solution is stable and unstable and determine all bifurcation points. Construct a bifurcation diagram plotting V vs. I_{app} of stable and unstable solutions, steady and periodic. Determine any I_{app} ranges where the model displays bistability of solutions. Provide phase plane plots (V vs. h) of the different types of solutions found.

4. Investigate a two-cell network consisting of two oscillating cells (modeled by the Morris-Lecar model) with reciprocal excitatory synaptic connections between them. Determine stable firing patterns when the individual cells fire at the same and different frequencies. Explain the effects the cells have on each other and why the observed stable firing patterns are generated using phase plane analysis.
5. Investigate a two-cell network consisting of two oscillating cells (modeled by the Morris-Lecar model) with reciprocal inhibitory synaptic connections between them. Determine what activation and decay rates of the synaptic gating variable give anti-phase oscillations. Can you determine rates that give in-phase oscillations? Explain the effects the cells have on each other and why the observed stable firing patterns are generated using phase plane analysis.
6. Investigate a two-cell network consisting of two identical cells (modeled by the Morris-Lecar model) with reciprocal inhibitory synaptic connections between them and with

electrical coupling. Look at cases when synaptic inhibition is weak compared to the electrical coupling and vice-versa. Determine stable firing patterns when one cell is oscillating and the other is not. Explain the effects the cells have on each other and why the observed stable firing patterns are generated using phase plane analysis.