Nonlinear Partial Differential Equations

(the course proposed to be offered at UNCBPA, Spring 2006)

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Office Hours: to be announced

Basic Text:
J. D. Logan, *An Introduction to Partial Differential Equations*, Wiley.
Additional Texts:
Lin and Segel:*Mathematics Applied to Deterministic Problems in the Natural Sciences*, Siam;
I. Stakgold: *Boundary Value Problems of Mathematical Physics*, Vol. I and II; Siam;
J. Kevorkian, *Partial Differential Equations*, Springer.
Selected research papers to be provided.

Tentative Course Outline:

Review of ordinary differential equations. Green's function methods, overview of the theory of distributions; δ function. Boundary value problems. Singular problems. Applications and examples.

Review of linear PDE's. Classification and canonical form. Elliptic, hyperbolic, parabolic problems. Examples: Laplace Equation, Wave Equation, Diffusion Equation. Fundamental solutions. General techniques for solving linear equations.

The Derichlet and Neumann problems. Free space point dipole solution. Surface layers. Solutions in finite and infinite domains. Examples from electromagnetism.

Hyperbolic problems. First order equations and characteristics. Cauchy-Kovalevski Theorem.

The concept of Weak solution. Application to shock waves. Burgers equation.
Student presentation I.

Hyperbolic systems and applications to gas dynamics and shallow water flow.

Diffusion equation revisited. Similarity methods. Nonlinear diffusion. Reaction-diffusion and perturbative methods. Fisher's equation. Applications in biological systems.

Convection-diffusion. Asymptotic solutions to Burgers' equation. Applications in physics.

Nonlinear elliptic equations. Weak and strong maximum principle. Eigenvalue problems. Examples of nonlinear elliptic equations in physics.

Student presentation II.

Introduction to fluid mechanics as an example of a nonlinear system of PDE's. Incompressible flow, Navier-Stokes equations. Potential flow problems.

Introduction to Regular perturbation theory. Examples from mechanics.

Asymptotic expansions and applications. Examples from thin film flows, including lubrication theory.
Student presentation III.

Review.