Math 637, Spring 2005, Study Questions/Problems

1. For a coupled system of chemical reactions (described by a set of ordinary differential equations), what is the difference between an equilibrium approximation and a quasi-steady approximation? Which is easier to use and why?

2. How is the velocity of an enzyme reaction (dP/dt) affected by competitive versus allosteric inhibition?

3. What are the different types of mass transport across a membrane, and which require an energy input (ATP)?

4. For a single ion type (e.g, Na^+), what equation gives the equilibrium elecrostatic membrane potential? Is the value unique, and what if there are multiple ion types?

5. What can we say about voltage-current relations for single and multiple ion types?

6. What is the difference between the instantaneous and steady-state voltage response of an ion channel?

7. What are the 4 dependent variables in the Hodgkin-Huxley model of an action potential?

8. What are some of the important assumptions/simplifications in the Hodgkin-Huxley model?

9. How does radius affect flow resistance in large blood vessels (for fixed radius)? How is this relation changed when compliance is included?

10. What are the various elements that might be included in a model of the circulatory system?

11. How is local blood flow regulated in response to metabolic demand?

12. What is partial prssure of a gas dissolved in a liquid, and how is it related to concentration (e.g., moles/volume)?

13. What is the Hb-O_2 saturation curve? How is it affected by pH, and how might this affect oxygen uptake and delivery by the blood?
14. In terms of modeling, what are the main differences between O_2 uptake and CO_2 release in the lungs? What about O_2 uptake in the lungs versus O_2 delivery to tissue?
15. Which class of blood vessels delivers the most O_2, especially in working muscle and heart, and why does this make sense?
16. What is the Krogh model of oxygen delivery?
17. What aspects of microvascular oxygen delivery and consumption does the Krogh model neglect?
18. What are the main effects seen in microvascular blood flow? How are these explained by the properties of blood flowing in small tubes?
19. What are the conservation equations needed to model two-phase continuum blood flow in the microcirculation? What empirical relations are also needed?

1. Consider a set of reactions where two enzymes compete for one of the same substrates:

$$A + B + E_1 \rightleftharpoons^{k_1}_{k_{-1}} C_1 \rightarrow^{k_2} E_1 + P_1$$

$$A + E_2 \rightleftharpoons^{k_3}_{k_{-3}} C_2 \rightarrow^{k_4} E_2 + P_2$$

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   (a) Use the equilibrium approximation and conservation of enzymes ($e_1 + c_1 = e_{10}$, $e_2 + c_2 = e_{20}$) to find the concentrations of C_1 and C_2.
   (b) Write down differential equations for the concentrations of C_1, C_2, A, and B.
   (c) Use the quasi-steady approximation and conservation of enzymes to find the concentrations of C_1 and C_2.

2. Consider given intra- and extra-cellular concentrations of Na^+ and K^+: Na_i, Na_e, K_i, and K_e. For fixed temperature T, what are the Nernst potentials V_{Na} and V_K?

   If simple linear I-V models are used, i.e., $I_{Na} = g_{Na}(V - V_{Na})$ and $I_K = g_K(V - V_K)$, where V is the membrane potential, what is the equilibrium (no net current) value $V = V_{eq}$?

   For $g_{Na} = 120mS/cm^2$, $g_K = 36$, $Na_i = 50mM$, $Na_e = 437$, $K_i = 397$, and $K_e = 20$, $T = 30^oC$, and $R/F = 0.000086mV/^oK$, what is V_{eq} and what are the Na and K currents?

3. Consider the simple, linear model of the circulation consisting of a single heart chamber, compliant systemic arteries and veins, and systemic capillaries. Describe the parameters and their effect on the solution for total blood flow:

$$Q = \frac{FC_dV_0}{C_v + (1 + FC_dR_s)C_a}$$

4. Consider the following simple model of microvascular blood flow distribution at diverging bifurcations (branchings), where the parent vessel has flow Q and radius R, and the daughter vessels have flows Q_1 and Q_2 and radii R_1 and R_2. The flow in each daughter is given by:

$$Q_i = Q * R_i^2/(R_1^2 + R_2^2) \quad (i = 1, 2)$$

   If the branching is asymmetric such that $R_1/R = 0.8$ and $R_2/R = 0.7$, what is the minimum flow after 1 branching, relative to the original parent flow? What about after 3 branchings? If average blood velocity is given by $v = Q/(\pi R^2)$, what is the minimum velocity after 3 branchings (relative to initial)?

   Now, in addition to the above total flow model, consider a simple model of red blood cell distribution at diverging bifurcations, where the parent vessel has RBC flow F and the daughter vessels have RBC flows F_1 and F_2:

$$F_i = F * \left[0.63(Q_i/Q - 0.5)^{1/3} + 0.5\right] \quad (i = 1, 2)$$

Using the radius branching ratios given above, what is the minimum red cell flow fraction after 1 branching? If we define discharge hematocrit as $H_D = F/Q$, how much is H_D decreased after 1 branching?