Math 450H
First Order Linear ODE Modeling Problems
Prof. Bukiet

1. **Chemical reactor / Mixture**. (Boyce and DiPrima 8e, p. 8, #21) A pond initially contains 1,000,000 gallons of water and an unknown amount of an undesirable chemical. Water containing 0.01 grams of this chemical per gallon flows into the pond at the rate of 300 gallons / hour. The mixture flows out at the same rate. Assume the pond is well-mixed. Write a differential equation for the amount of the chemical in the pond at any time. How much of the chemical will be in the pond after a very long time?
Set up the same problem where the mixture flows out at the rate of 400 gallons / hr.

2. **Amount of a drug in one's system**. (Boyce and DiPrima, p. 8, #24) A certain drug is being administered intravenously to a hospital patient. Fluid containing 5 mg/cm^3 of the drug enters the patient's bloodstream at a rate of 100 cm^3/hr. The drug is absorbed by the tissues or is removed from the bloodstream at a rate proportional to the amount in the bloodstream, with rate constant 0.4/hr. Assume that the drug is uniformly distributed in the bloodstream. Write a differential equation for the amount of drug present in the bloodstream at any time. How much is present in the bloodstream after a long time?

3. **Radioactive Decay**. (Boyce and DiPrima, p. 17, #12) A radioactive material, such as the isotope thorium-234, disintegrates at a rate proportional to the amount currently present. If 100 mg of thorium-234 decays to 82.04 mg in 1 week, determine the decay rate. Find an expression for the amount of thorium-234 present at any time.

4. **RC-circuit**. (Boyce and DiPrima, p. 18, #17) Consider an electrical circuit containing a capacitor, resistor and battery/ The charge staisfies the equation

$$R\frac{dQ}{dt} + \frac{Q}{C} = V$$

where R is the resistance, C is the capacitance and V is the constant voltage supplied by the battery. Find the charge as a function of time and the limiting (long-time) charge.

5. **Continuous compounding**. (Boyce and DiPrima, p. 61, #9) A person borrows \$8,000 to buy a car. The lender charges interest at an annual rate of 10%. If interest is compunded continuously and payments are made continuously at the rate of \$k per year, find k such that the loan is paid off in 3 years.

6. **Free fall**. (Boyce and DiPrima, p. 64, #20) A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/sec from the roof of a building 30 m high. (Neglect air resistance). Find the maximum height above the ground that the ball reaches.

6a. **Free fall with linear air resistance**. (Boyce and DiPrima, p. 64, #21) See previous problem, but air resistance is $|v|/30$ kg m/sec^2

7. **Free fall - Baseball**. Let $v(t)$ and $w(t)$ be the horizontal and vertical components of velocity of the ball hit in a baseball game. Neglecting air resistance, we have $\frac{dv}{dt} = 0$ and $\frac{dw}{dt} = -g$. Thus, $v = u\cos A$ and $w = -gt + u\sin A$, where u is the initial speed of the ball and A is the angle of elevation. Find $x(t)$ and $y(t)$, the horizontal distance and vertical height. If the wall is distance L from home plate and is H feet high, find a relation between u and A that must be satisfied for the ball to clear the wall. How would the model equations differ if there were air resistance?