Math 450H
Homework VI: Due 12/8/05

Prof. Bukiet

Topic: Electrostatic potential

1. Consider the 2D Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

for the electrostatic potential ϕ in a rectangular domain of size $7\ in(x) \times 9\ in(y)$, where the Boundary Conditions are one of the following:

(a) $\phi = 0$ at $y = 0$ and $y = 9$, and $\phi = 12$ at $x = 0$ and $x = 7$

(b) $\phi = 0$ at $x = 7$ and $y = 0$, and $\phi = 12$ at $x = 0$ and $y = 9$

Choose **one** of the above boundary conditions and obtain the theoretical solution for this problem using separation of variables and the superposition principle, similar to what was done in class. Sum the resulting series numerically. Include the terms in the series with absolute value $> 10^{-9}$ (how many is this?). Also, calculate the electric field as the negative gradient of the potential.

Plot the results for the potential and the electric field. In particular, make a contour plot (e.g., using Matlab) of the potential using (labeled) contours at $\phi = 0, 1, 2, ..., 12$.

2. Use "Equipotential and Field Mapper" to find the electrostatic potential in a $7\ in(x) \times 9\ in(y)$ rectangular domain bounded by four electrodes, with imposed boundary conditions matching those used in the theoretical solution above. For example, for Boundary Condition (b) two electrodes at $x = 0$ and $y = 9$ are connected to the battery (approx. 12 V), and the other two are kept at zero potential.

Find and sketch equipotential lines (lines of equal potential) in $2\ V$ intervals. Find and sketch similar number of electric field lines (noting that the gradient must be perpendicular to level curves.

Compare the theoretical and experimental results and discuss any differences, in particular close to the corners.