Math 450H, Fall 2004
Final Optional/Extra Credit HW
Due 12/07 Prof. Bukiet

Numerical Solution of Laplace's Equation in a Rectangle

We will have discussed 3 methods of solving Laplace's equation numerically on a rectangle in class. In this homework/project, work in groups of 3. Each one in a group will solve the Laplace equation given below using one of the methods below. Then, as a group, you'll compare the solutions and comment on them.

Solve the Laplace equation in a rectangular domain using the finite difference method. Use a domain of size 7×9 inches, and the following boundary conditions: $\phi = 0$ at $y = 0$ and $x = 7$, and $\phi = 12\ V$ at $x = 0$ and $y = 9$.

Use grid spacing of 1 inch in each direction.

- Method 1: Solve the linear algebra finite difference problem using Gaussian elimination.
- Method 2: Solve using Jacobi's method and then using the Gauss-Seidel method. Comment on your stopping criterion and the speed of convergence.
- Method 3: Use the tour du wino method discussed in class and in Farlow's text. Clearly describe your work and your stopping criterion.

Plot the results or present them in a nice way so that one can compare how close the answers are. Should they all be the same? Why or why not? What issues are involved? Perform this experiment in the Capstone Lab using the Electrostatic Field Mapper equipment to further check your results. Present this result along with the numerical results. Comment on the relative merits of each numerical method, its advantages and disadvantages.

Choose one of the methods on which to solve with a refined grid. Refine the grid enough so that you are confident that the solution is within 0.01 of the correct values at each grid point. Discuss the results and your thoughts on the speed of convergence.

References: Farlow "PDE's for Scientists and Engineers"; Lecture notes; Books of your choice.