

Math 450H
Homework IV: Due 11/11/04
Prof. Bukiet

1. Consider the spring pendulum, as explained in class.

Note that you should have a Runge-Kutta code for plain mass-spring computations. Call this computer code 1. Write out the system of 2 first order ODEs.

Note that you should have a Runge-Kutta code for plain pendulum computations. Call this computer code 2. Write out the (nonlinear) system of 2 first order ODEs.

Extend your numerical code from Homework 3 to the system of 4 nonlinear ODE's governing the dynamics of a spring pendulum (without damping) using the r , θ (i.e., polar) formulation. Call this computer code 3.

Also, write a code using the Cartesian formulation. This should be a nonlinear system of 6 first order ODEs. Call this computer code 4.

For the parameters, use $L_0 = 10$ cm (length from pivot to end of spring without weight), $g = 1000$ cm/s, and k/m as specified below. Do the following:

1. *Limit of zero θ .* Assume $\theta(0) = 0$, $\dot{\theta}(0) = 0$, $l(0) = l_{eq} + 1.0$ cm, $\dot{l}(0) = 0$, $k/m = 100$ s⁻², where l_{eq} is the equilibrium length of the spring (with the mass on it). Integrate the equations for at least one period of a simple mass on a spring system (code 1). Confirm that codes 3 and 4 give the same results.

Plot $l(t)$ and $\dot{l}(t)$ for any of the codes to make sure that the path in the phase plane is as it should be (ellipse).

2. *Limit of "infinite" spring constant.* Now, use $\theta(0) = 0.1$ radians, $\dot{\theta}(0) = 0$, $l(0) = l_{eq}$, $\dot{l}(0) = 0$, $k/m = 10000$ s⁻². Integrate the equations for at least one period of a simple pendulum with codes 2, 3, and 4. Confirm, using a table or by making plots that all 3 give the same result. (Well, not quite the same since k/m is not infinite). You may have to adapt the code 4 output to provide the appropriate variables for comparison. Make one phase plane plot of $\theta(t)$, $\dot{\theta}(t)$. Is the motion in the θ direction periodic? l should be pretty much constant if the spring constant is very high. Verify this with the code 3 output.
3. *General case.* Now, use $\theta(0) = 0.1$ radians, $\dot{\theta}(0) = 0$, $l(0) = l_{eq} + 1.0$ cm, $\dot{l}(0) = 0$, $k/m = 100$ s⁻² (this value of k/m is realistic for our lab system). Use code 3 to compute θ vs. length for time 0 through 10 seconds. Use code 4 to compute z vs. x (where z is the vertical position and x is the horizontal position of the mass) for time 0 through 10 seconds.