Math 450H
Homework II: Due 10/10/05
Prof. Bukiet
Topic: Calculus of Variations and the Brachistochrone Problem

1. In class, we analyzed the brachistochrone experiment and performed it with the Logger-Pro setup. Do the experiment with a partner five times each for the cycloid and the line. Compare your results with theoretical times and relative theoretical times. (I.e., do 5 experiments and average the results. This should reduce experimental errors.)

2. Consider an inverted cycloid (or brachistochrone shape) (for $\omega = 0$ to $\omega = \pi$, corresponding (0,0) to (a, b)) versus a line with the same endpoints. Find the length of the linear path, the length of the cycloid path and the ratio of these lengths. In your experiments, find these lengths and use your experimental timing results to find the average velocity (path length / time) for the 2 cases and compare with theoretical values.

3. For the path of a line from (0,0) to (a, b) with zero initial velocity, find the theoretical time (without friction) to traverse that path. Find the average speed (path length / time) for the trip and say something interesting about the result.

4. How do you know which cycloid to use given the points (0,0) and (a, b)?
Convince yourself that there is not necessarily a cycloid (from 0 to π) for any a and b. Let $x = r(\omega - \sin\omega)$ and $y = r(1 - \cos\omega)$ on $0 \leq \omega \leq 2\pi$. The point (a, b) at the end of the portion of the cycloid usually corresponds to a value of ω that is not π, call it Ω. Compute the appropriate values of r and Ω for the following cases:
a. (a, b)=(1,4)
b. (a, b)=(1,2)
c. (a, b)=(1,1)
d. (a, b)=(1,1/2)
e. (a, b)=(1,1/4)
by performing the following steps.
A. Divide y by x (or vice versa) to eliminate r and get an equation in one variable, Ω.
B. Use Newton's method for each of a.-e. to compute Ω. (Fully explain your code and derive the relevant equations – iteration scheme).
C. Once you compute each Ω just plug in for r.
Make a table of your values of Ω and the corresponding values of r. (Have 3 columns with one being b/a or a/b.
Extra Credit: Can you guess or show what value of Ω minimizes r

5. Derive (using calculus of variations) the ordinary differential equation for the extremal of the surface area of revolution of a function $f(x) > 0$ from $(-\ln 2, 5/4)$ to $(\ln 2, 5/4)$ around the x-axis. Show that the catenoid $y = \cosh(x)$ is a solution to this differential equation.

Your report must have clear explanations, including paragraphs and complete sentences. It must be typed. All work (experimental, analytical and computational) must be your own – except experimental results done with a partner.