

Math 450H
Homework I: Due 09/22/05
Prof. Bukiet

Topic: Math Modeling Basics and Newton's Law of Cooling

1. When brought to the emergency room, an accident victim has 3 liters of blood and is still losing blood at the rate of 0.25 liter per hour. The victim is immediately given continuous blood transfusions at 0.5 liter per hour, and an antibiotic is administered intravenously at 0.5 gm per hour. Over the next 4 hours, the bleeding stops (assume linearly, i.e. it's .125 liters per hour after 2 hours and 0 after 4 hours). Two hours after the bleeding stops, the infusion of blood and antibiotics is stopped. Determine the concentration of the antibiotic in the patient's blood when the transfusion stops. (Assume that the antibiotic is broken down or otherwise removed from the bloodstream proportionally to the amount (in grams) (NOT THE CONCENTRATION AS EARLIER CLAIMED) of the antibiotic in the bloodstream at any time such that half of a given amount infused into the bloodstream at a given instant remains 3 hours later.)

2. Take a styrofoam cup 3/4 filled with water and raise the temperature using a microwave to between 130 and 170 degrees Fahrenheit. Put the cup in another styrofoam cup (so you don't burn yourself and to insulate the experiment). Record the ambient temperature. Cover the top of the cups and stick the temperature probe into the water. Record the temperature every 2 minutes until the water is approximately 20 degrees above room temperature. For all parts of the problem, we will use the measured ambient temperature and the measured initial temperature.

(a). Use Newton's Law of Cooling as your model to compute the value of the "constant" k using least squares on the \log of the experimental and analytical temperature difference from ambient. Compute the "average square error".

(b). Compute k values numerically to minimize the average absolute error and the average square error.

(c). Use consecutive points to investigate how the value of k changes with temperature and plot this curve. Does k appear to be constant?

(d). Allow the Law of Cooling exponent p to be different from 1.

$$\frac{dT}{dt} = -k(T - T_A)^p$$

Compute the best values of k and p to minimize error in temperature. (Hint: You will probably want to use a \log formulation, but try to be creative and logical). Describe clearly the steps you take and why you feel your strategy is justified. You might compare your results with other approaches to support your method.

(e). Perform steps (a)-(d) for the same experiment with either a different liquid (make sure it does not burn) or with water constantly stirring the liquid and without the top and in a wider container. (You might want to take more frequent measurements).

Your report must have clear explanations, including paragraphs and complete sentences. All work (experimental, analytical and computational) must be your own.