Math 111-H03, Problems, 7 Dec 2007

Be sure to carefully show all work. Hand in Problems 1, 4, 6, and 7.

Problem 1 Find

a) $\lim_{x\to 0}(x+1)^{\cot x}$

b) $\lim_{x\to 0}\frac{\sin 2x+\tan x}{3x^2}$

c) $\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}$

d) $\lim_{x\to a}\frac{a-x}{\ln\frac{x}{a}}$

Problem 2 The sum of two nonnegative numbers is 10. Find the minimum possible value of the sum of their cubes.

Problem 3 Find the dimensions of the right circular cone of maximum volume inscribed in a sphere of radius r.

Problem 4 A soccer goal is 8 meters wide. Gerardo runs along a straight line that is 8 meters to the side of the goal parallel to the long side of the soccer field. At which point should he kick the ball?

Problem 5 Conical paper cups are made so that their depth is $\sqrt{2}$ times the radius of the rim. Show that this design requires the least amount of paper per volume.

Problem 6 Which sector shoudl be cut out from a disk of radius R, so that the funnel made from the remaining part has the largest possible volume?

Problem 7 Use Riemann Sums to Evaluate the integral $\int_1^2 \frac{dx}{x^2}$. *Hint:* Use $x_i^* = \sqrt{x_{i-1}x_i}$ as sample points.