

Math 111-H03, Problem Set I, Fall 2007

Hand in Problems 2,4-7 on Friday 21 September 2007

Problem 1 Sketch the graph of the following functions:

a) $f(x) = \frac{1}{|x-5|}$ b) $h(y) = 2^{\ln(3y-2)}$ c) $g(t) = \cos(2t)$
d) $\sin(|x|)$ e) $f(x) = \cos^2(\sin^{-1}(x))$

Problem 2 Suppose $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$. What is

a) $f(g(x))$ and b) $g(f(x))$? (Simplify)
c) What is the domain of $f \circ g$? $g \circ f$?

Problem 3 A variable star is a star whose brightness increases and decreases periodically. For Delta Cephei, one of the most visible variable stars, the time between periods of maximum brightness is 5.4 days. The average brightness is 4.0 and its brightness varies by ± 0.35 . Find a function that models the brightness of Delta Cephei as a function of time.

Problem 4 a) If $f(x) = 3x + 2 \ln x$, find $f^{-1}(3)$.

b) Find the inverse of $f(x) = \frac{1+2^x}{1-2^x}$

c) Suppose g is an even function. Is $f \circ g$ always an even function?

Problem 5 Consider the function given by

$$\begin{aligned} f(x) &= -1, x < 0 \\ &= 2, x = 0 \\ &= x^3, x > 0 \end{aligned} \tag{1}$$

a) Sketch the graph of the function.

b) Evaluate the following limits (if they exist): (i) $\lim_{x \rightarrow 0^+} f(x)$, (ii) $\lim_{x \rightarrow 0^-} f(x)$, (iii) $\lim_{x \rightarrow 0} f(x)$, (iv) $\lim_{x \rightarrow 1} f(x)$ (v) $\lim_{x \rightarrow -1} f(x)$

Problem 6 Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{2-x}-1}{\sqrt{5-x}-2}$

Problem 7 Is there a number b so that $\lim_{x \rightarrow 1} \frac{3x^2+bx+4}{x^2-1}$ exists? if so, find the value of b and the value of the limit.

Problem 8 (Extra Credit!) Find the equation of a line that is normal to $y + 2x = 0$ and tangent to $y = x^2$.