Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (9 points) Differentiate

$$(a)\ f(x) = e^{-4x}\sin 3x, \qquad (b)\ g(x) = \frac{x(2+x)}{3-x}, \qquad (c)\ h(x) = \sqrt{4+\sqrt{3+x}}.$$

2. (9 points) Compute the following limits (possibly infinite):

$$(a)\ \lim_{x\to+\infty} \frac{2x^{5/2}+x^{3/2}+1}{3x^{5/2}+x^{3/2}+1}, \qquad (b)\ \lim_{x\to-3^-} \frac{x^2+1}{2x+6}, \qquad (c)\ \lim_{x\to 0^+}\Big(\ln(\sin x) - \ln x\Big).$$

3. (9 points) Use L'Hospital's Rule to compute the following limits:

$$(a)\ \lim_{x\to+\infty} \frac{x(2-x^2)}{x^3+1}, \qquad (b)\ \lim_{z\to 0} \frac{z^2}{\ln(\cos z)}, \qquad (c)\ \lim_{\theta\to 0}(1+\theta)^{\cot\theta}.$$

4. (10 points) Let $f(x) = \sqrt{x}$.
   (a) Use the definition of derivative to compute $f'(x)$ for $x > 0$.
   (b) Use the definition of derivative to show that $f'(x)$ does *not* exist at $x = 0$.

5. (7 points) Find an equation for the line tangent to the curve $y = \arcsin x$ at $x = \frac{1}{2}$. Draw a sketch.

6. (7 points) Two airplanes are flying at the altitude of 12 km along straigh line courses that intersect at a right angle. Plane A is approaching the the point of intersection at 700 km/h. Plane B is approaching the intersection at 900 km/h. At what rate is the distance between the planes changing when plane A is 5 km away from the intersection and plane B is 12 km away from the intersection?

7. (18 points) For the function
$$f(x) = \frac{x^2+5}{x-2},$$
find the following, if they exist: (i) all local extrema, (ii) intervals where the function increases or decreases, (iii) all points of inflection, (iv) intervals of upward or downward concavity, (v) all asymptotes (vertical, horizontal, slant, if any). Sketch a plot of the curve $y = f(x)$.

8. (7 points)
   (a) Find the point Q on the line $y = 2x$ that is closest to the point $P(1, -3)$.
   (b) Show that the line passing through P and Q is normal to the line $y = 2x$.

9. (7 points)

   (a) Find dy/dx by implicit differentiation, if

   $$x^5 + y^5 = 15x^2y^2.$$

   (b) Find the antiderivative $F(x)$ of $f(x) = e^x + \sin 2x - 3$ that satisfies $F(\pi) = 5$.

10. (8 points) The velocity v as a function of time t of a locomotive reversing its direction of motion is given in the following table:

t (s)	0	2	4	6	8	10
v (m/s)	7.8	6.2	0.8	0.	-4.4	-5.6

   (a) Assuming that $v(t)$ is a monotonically decreasing function, give the upper and lower estimate of the distance traveled by locomotive before stopping, based on the above data.

   (b) Use the Midpoint Rule to estimate the total distance traveled by the locomotive during the time interval from 0 to 10 seconds, assuming $v(t)$ is linear on each of the subintervals.

11. (9 points)

   (a) Express the limit

   $$\lim_{n\to\infty} \sum_{i=1}^{n} (3x_i^2 - 2x_i + 1)\Delta x, \qquad [-1, 2],$$

   as a definite integral. Here x_i's are the right end-points of the subintervals of equal length Δx partitioning the indicated interval.

   (b) Evaluate this limit.

   (c) Interpret this limit geometrically by drawing a sketch and explaining the geometric meaning of the answer in part (b).

$$\left[Hint: \ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \right].$$