Math 111H **Exam 3** November 16, 2005

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (15 points) Use l'Hôspital's Rule to compute the following limits

$$(a)\ \lim_{t\to\infty}\frac{t^3-1}{4t^3-t-3}, \qquad (b)\ \lim_{\theta\to 0}\frac{\sin(\theta^2)}{\theta}, \qquad (c)\ \lim_{x\to 0}\frac{x(1-\cos x)}{x-\sin x}.$$

2. (13 points) Use implicit differentiation to find an equation of the line tangent to the curve

$$12(x^2+y^2)=25xy,$$

at point $(3,4)$. Verify that this point lies on the curve.

3. (13 points)

   (a) Compute dy/dx for the following function, using logarithmic differentiation:

$$y=\sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}.$$

   (b) For $y=2\cosh(3x)-4\sinh(3x)$, compute d^2y/dx^2 and express your answer in terms of y only.

4. (13 points) The angle of the elevation of the sun is 45 deg ($=\pi/4$ rad) and is decreasing at 1/4 rad/hr. The NJIT flag pole is 50 ft tall. How fast is the shadow of the pole lengthening, assuming that the shadow is cast on level ground?

5. (13 points) Find the absolute minimum and maximum of the function $f(x)=1+\sqrt[4]{(x+1)^2}$ over the interval $[-2,3]$.

6. (20 points) For the function $f(x)=1-9x-6x^2-x^3$, find: (i) all local minima and maxima, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity. Sketch the graph of the function.

7. (13 points) Use linear approximation to estimate $\cos(29^\circ)$.