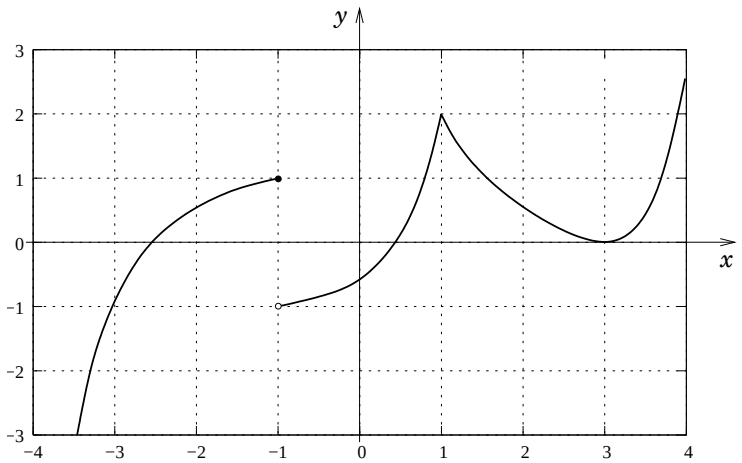
Math 111H **Exam 2** October 19, 2005

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (16 points) The graph of $f(x)$ is shown below.



   (a) Find: (i) $\lim_{x\to 1} f(x)$, (ii) $\lim_{x\to -1^+} f(x)$, (iii) $\lim_{x\to -1^-} f(x)$.

   (b) At what points is $f(x)$ continuous?

   (c) Find $f'(3)$. At what points is $f(x)$ differentiable?

   (d) Sketch the graph of $f'(x)$.

2. (15 points) Compute the following limits:

$$(a)\ \lim_{x\to\infty} \frac{x^3 - 3x^2 + 2}{1 - x + 5x^3}, \qquad (b)\ \lim_{x\to -\infty} \frac{3x}{\sqrt{x^2 - 4}}, \qquad (c)\ \lim_{x\to 0} \frac{\sin 4x}{\tan 7x}.$$

3. (12 points) Find the constant c so that the following function is continuous:

$$f(x) = \begin{cases} \frac{\sqrt{1+cx}-1}{x}, & x < 0 \\ f(x) = \cos(x) + c, & x \geq 0. \end{cases}$$

4. (15 points) Differentiate the following functions:

$$(a)\ \frac{x^3 - x^2}{x^2 + x - 2}, \qquad (b)\ \cos^2 x - \frac{1}{x^{3/2}}, \qquad (c)\ xe^{2x}\sin(3x).$$

5. (12 points) Use the definition of the derivative to find $f'(x)$, if

$$f(x) = \frac{1}{\sqrt{2x}}.$$

6. (15 points) A particle moves according to the law $s = f(t)$, where $f(t) = t^3 - 6t^2 + 9t + 5$.

   (a) Find the velocity $v(t)$.

   (b) When is the particle at rest?

   (c) When is the particle moving in the positive direction?

   (d) Find the distance traveled during the first 5 seconds.

   (e) Draw a diagram to illustrate the motion of the particle.

7. (15 points) During the period between 1790 and 1910, the population of the US was seen to be roughly described by

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)\exp(-kMt)}$$

   where t is the time (in years) elapsed since 1790, P_0 is the US population in 1790, M and k are constants.

   (a) Find the rate of population growth at time t.

   (b) Show that $P(t)$ satisfies the following equation:

$$\frac{dP}{dt} = kP(M - P).$$

   (c) What happens to the population as $t \to \infty$?

   (d) What happens to the rate of population growth as $t \to \infty$?