Math 222, Fall 2007
Matlab Project #1: Euler's Method
Due date: October 12, 2007

Write a matlab script that uses Euler's method to integrate the following initial value problem (IVP) from $t = 0$ to $t = 1$:

$$y' = 2y - 1, \qquad y(0) = 1.$$

Your script should include the following features:

- You should be able to specify the number of steps N from $t = 0$ to $t = 1$, from which your script calculates the step size $h = 1/N$.
- Your script should store the values of t_n and the numerically obtained values of y_n for $n = 1, \ldots, N$ (note that $t_0 = 0$ and $t_N = 1$).
- Your script should allow you to input the exact solution of the initial value problem (IVP) for $y(t_n)$.
- Your script should calculate and store the cumulative error at each step, according to the formula $e_n = |y_n - y(t_n)|$.

Using your script, do the following:

1. Find the exact solution of the IVP and enter it into the appropriate part of your script.
2. Run your script for $N = 16$, $N = 32$, $N = 64$, etc., doubling the number of steps for each run up to $N = 8192$, in each case storing the final cumulative error value, e_N. Graph these error values versus the corresponding step sizes h. Deduce an approximate formula for how the cumulative error depends on h. Explain why we refer to Euler's method as being *first-order accurate*.
3. Run the script with your smallest value of h and, on the same axes, graph the numerically obtained solution using circles ('o') and the exact solution using a solid line.

Format. Your project must be typed on a computer, with the exception of equations, which may be written into blank spaces left in the typed text. All graphs must have a title and axis labels (type `doc plot` at the matlab command line for instructions). Remember to explain your results and to provide sufficient graphical evidence to support them. Include a printout of the script in your project.