Sample Exercise for the Supplement on Algebraic Errors and Practice to Avoid Them.

Instructions: The following computations contain algebraic errors. Find the errors and state the Algebraic Rule that has been violated. Then do the computation correctly.

1. Expand $(x+5)^2$.

Incorrect Solution: $(x+5)^2 = x^2 + 5^2$

$$= x^2 + 25$$

Response: The computation violates the rule: Integer Exponents distribute over factors, not terms, or equivalently Integer Exponents distribute over products not sums.

Correct Computation: $(x+5)^2 = x^2 + 10x = 25$ using the rule that when squaring an expression containing two terms (a binomial) we use the rule, "first term squared and the product doubled plus the last term squared.".

2. Solve for x : $x^2 + 3x = 4$

Incorrect Solution: $x^2 + 3x = 4$

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$$x(x+3) = 4$$

$$x = 4 \ \ or \ \ x + 3 = 4$$

$$x = 1$$

Response: The computation attempts to use the "Zero Product Rule " incorrectly. The rule states that if a product of two factors equals zero then the equation will be satisfied if either factor is zero. In the above computation we observe a product is equal to 4 and not zero, if we set each factor equal to 4 we may or may not get correct answers. In the above case x=1 actually works, but the other answer x= 4 does not work.

Correct Computation: $x^2 + 3x = 4$

$$x^2 + 3x - 4 = 0$$

$$(x-1)(x+4) = 0$$

$$x - 1 = 0 \ \ or \ \ x + 4 = 0$$

$$x = -4$$

After factoring we had a product of two factors being equal to zero, which allowed us to use the "Zero Product Rule " and set each factor equal to zero.

3. Simplify: $\frac{6x+2}{2}$

Incorrect Solution. $\frac{6x+2}{2} = \frac{6x+2}{2}$

$$= 3x + 2$$

Response: The computation violates the rule that cancellation in a fraction can only take place between a factor of the entire numberator of the fraction and a factor of the entire denominator of the fraction. In the above computation we cancelled into the 6 which is only a factor of the first term in the numerator.

Correct Computation: $\frac{6x+2}{2} = \frac{2(3x+1)}{2}$

$$= 3x + 1$$

In the correct computation 2 was a factor of the entire numerator of the fraction and it can be considered a factor of the entire denominator of the fraction, so cancellation was permissible.

Alternative Correct Computation: $\frac{6x+2}{2} = \frac{6x}{2} + \frac{2}{2}$
$$= 3x + 1$$

In the alternative correct computation the denominator, "2" divides EACH term in the numerator.

4. Solve for x : $\sqrt{x^2+9} = 5$

Incorrect Solution: $\sqrt{x^2+9} = 5$
$$x + 3 = 5$$
$$x = 2$$

Response: the incorrect solution violates the rule that : Square roots distribute over positive factors but not terms, or equivalently square roots distribute over products of positive factors but not over sums. Note we will avoid square roots of negative numbers because working in the complex number system is beyond the scope of this course; but for those who know a little observe: $[(-1)(-1)]^{\frac{1}{2}} = (1)^{\frac{1}{2}} = 1$,

but $(-1)^{\frac{1}{2}}(-1)^{\frac{1}{2}} = (i)(i) = -1$ so one must learn a good deal more before working in the complex number system.

Correct Solution $\sqrt{x^2+9} = 5$, squaring both sides of the equation gives
$$x^2 + 9 = 25$$
$$x^2 = 16$$
$$x = \pm 4$$

5. Solve for R, $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$

Incorrect Solution: $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$
$$R = x + y$$

Response: The incorrect solution violates the rule that when taking reciprocals of each side of an equation, each side must be treated as a single fraction, in this example the right side of the equation should be made into a single fraction before taking reciprocals.

Correct Solution: $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$
$$\frac{1}{R} = \frac{y+x}{xy}$$
$$R = \frac{xy}{y+x}$$

6. Simplify the expression $\frac{3x^{-1}+4}{x}$ to an expression without any negative exponents.

Incorrect Computation: $\frac{3x^{-1}+4}{x} = \frac{3+4}{x^2} = \frac{7}{x^2}$

Response: The incorrect solution violates the rule that a factor of the numerator of a fraction may be moved to the denominator of <u>that</u> fraction (THE FRACTION WHOSE NUMERATOR IT IS A FACTOR OF), if the sign of its exponent is changed.

Correct Computation: In the espression $\frac{3x^{-1}+4}{x}$, since x^{-1} is a factor of the first term (only) we think of the first term as $\frac{3x^{-1}}{1}$. Since x^{-1} is a factor of the numerator of this fraction we can move it to become a factor of the denominator of <u>this</u> fraction by changing the sign of its exponent. Thus we have $3x^{-1} = \frac{3x^{-1}}{1} = \frac{3}{x}$, continuing using this result gives $\frac{3x^{-1}+4}{x} = \frac{\frac{3}{x}+4}{x}$

$$= (\frac{\frac{3}{x}+4}{x})\frac{x}{x}$$

$$= \frac{3+4x}{x^2}$$

Working With Logarithms (This subject will be studied intensively in Math 108, but at this time we want to initiate the formation of habits that will help students avoid the most extremely common errors students make when working with logarithms.)

The function $ln(x)$ is called the natural logarithm of x. A very important number in mathematics, like the irrational numberr π which is approximately 3.14, is the number $e,$ which is approimately 2.72. For any positive number $x,$ $ln(x)$ is the power (exponent) which e must be raised to to get to $x,$ that is $e^{(ln(x))} = x.$ Specifically, $ln(e^2)$ is the power we must raise e to, in order to get e^2, so $ln(e^2)$ =2. At this time we are concerned with the following facts about logarithms:

RULES THAT EVERY STUDENT SHOULD KNOW ABOUT LOGARITHMS

a). The Log of Products Rule: $ln(xy) = lnx + lny$

b). The Log of Quotients Rule: $ln(\frac{x}{y}) = lnx - lny$

c). The Log of Powers Rule: $ln(x^p) = p(lnx)$

d). There is no Log of Sums rule, $ln(a+b)$ $is\ NOT$ equal to $lna + lnb$ and cannot in general be simplified.

e). There is no Power of Logs rule, $(lnx)^p$ $is\ NOT$ equal to $p(lnx).$ Be sure to observe the difference between $ln(x^p)$ for which there is a rule and $(lnx)^p$ for which there is no rule.

f). $(lnx)(lny)$ and $\frac{lnx}{lny}$ cannot in general be simplified .

Exercises:

7. If $lnx = a,$ and $lny = b$ Express $ln(x+y)$ in terms of a and $b.$

Incorrect Computation : $ln(x+y) = lnx + lny$
$= a + b$

Response: The above computation violates the fact that there is no Log of Sums Rule (see d above)

Correct Computation: if $ln(x) = a$ then a is the power we raise e to, in order to get $x,$ so $e^a = x,$ likewise if $ln(y) = b$ then $e^b = y,$ therefore $ln(x+y) = ln(e^a + e^b)$ and that is the best we can do.

8. If $lnx = a$ and $lny = b,$ Express $(lnx)(lny) - ln(xy)$ in terms of a and $b.$

Incorrect Computation: $(lnx)(lny) - ln(xy) = (lnx)(lny) - (lnx)(lny)$
$= ab - ab$
$= 0$

Response: The above computation replaces the second term $ln(xy)$ with $(lnx)(lny)$ but $ln(xy) = lnx + lny$, it is NOT equal to $(lnx)(lny)$.

Correct Computation: $(lnx)(lny) - ln(xy) = (lnx)(lny) - (lnx + lny)$

$= ab - (a+b)$

9. Solve for x if $x = ln(e^2)$+$(lne)^2$

Incorrect Solution $x = ln(e^2) + (lne)^2$

$= 2lne + 2lne$

note that since lne is the power that e must be raised to in order to get to e, $lne = 1$. $s$ince $e^1 = e$

therefore continuing we have $x = 2 + 2$

$x = 4$

Response: The incorrect computation violates the fact that there is no Power of a Log Rule (see e above) so the second term on the right side of the equation, $(lne)^2$ is not equal to $2lne$

Correct Solution: $x = ln(e^2) + (lne)^2$

$= 2lne + (lne)^2$

$= 2(1) + (1)^2$

$= 2 + 1$

$= 3$

Working with the exponential function, e^x

10. If $x = a + b + 2$ find e^x

Incorrect Computation: $x = a + b + 2$

$e^x = e^a + e^b + e^2$

Response: Before exponentiating each side of an equation (making each side of an equation the exponent to the base e) we bracket each side (if the side has more than one term) of the equation and if appropriate use the rule that $e^{(a+b)} = e^a e^b$. (the repeat the base and add the exponent rule in reverse) Note $e^{(a+b)}$ is NOT equal to $e^a + e^b$

Correct Computation: $x = a + b + 2$

$x = (a + b + 2\ )$

$e^x = e^{(a+b+2\ )}$

$= e^a e^b e^c$

11. Evaluate x^2 raised to the thrird power.

Incorrect Computation: $(x^2)^3 = x^{2^3} = x^8$

Response: The incorrectd computation violates the rule that $(a^b)^c = a^{(bc)}$

Correct Computation: $(x^2)^3 = x^{[(2)(3)]} = x^6$