Math 222 EXAM III, April 13, 2005

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (10) An object weighing 4lb (mass $m = \frac{1}{8}\text{lb}\cdot\text{s}^2/\text{ft}$) stretches a spring $\frac{1}{2}$ft. Determine the natural frequency and the period when the spring is set in motion.

2. (12) Find the Laplace Transform of the function $f(t) = t[1 - u_1(t)]$ **using the definition of the Laplace Transform**.

3. (8) Use the convolution theorem to find the inverse Laplace Transform of the function below. Do not evaluate the integral.
$$F(s) = \frac{1}{s^3(s^2+4)}.$$

4. (12) Consider the function $g(t) = t + \alpha u_1(t) - tu_3(t)$.
   (a) (4) Determine the value of the constant α for which $g(2) = 1$.
   (b) (8) Carefully sketch $g(t)$ on the interval $t \in [0, 5]$ using the value of α from part (a).

5. (12) Find the inverse Laplace Transform of the given functions.
$$(a)\ \ F(s) = \frac{1-e^{-s}}{s}; \qquad (b)\ \ H(s) = \frac{s+6}{s^2+6s+10}.$$

6. (20) An object weighing 64lb (mass $m = 2\text{lb}\cdot\text{s}^2/\text{ft}$) stretches a spring 8 ft. The undamped system, initially at rest, is suddenly set in motion by an external force of $12\sin(\omega t)$lb.
   (a) (5) Write the governing equation and initial conditions for this system.
   (b) (5) For what value of ω will the system exhibit resonance?
   (c) (10) Use the Laplace transform to solve the IVP with $\omega = 1$ (do **not** use the convolution theorem).

7. (8) Find the inverse Laplace Transform of the function
$$F(s) = \frac{e^{-2s}}{(2s+1)^2+4}.$$

8. (18) Consider the initial value problem
$$y'' + y = \gamma\delta(t - \pi/4), \qquad y(0) = 1, \quad y'(0) = -1.$$
   (a) (12) Find the solution of the given initial value problem.
   (b) (6) Determine the value of γ such that all motion ceases (i.e. $y = 0$) for $t \geq \pi/4$. (Useful fact: $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$)