Math 222 EXAM II, March 9, 2005

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (12) Solve the Initial Value Problem:

$$y'' + 4y = 6\sin(4t), \qquad y(0) = 0, \quad y'(0) = 0.$$

2. (12) Find the general solution:

$$y^{(4)} - 3y'' - 4y = t + 2.$$

3. (10) If the Wronskian of $f(t)$ and $g(t)$ is e^t, and if $f(t) = e^t$ and $g(0) = 0$, find $g(t)$.

4. (12) Determine (i) the complementary solution, and (ii) the form of a particular solution using the Method of Undetermined Coefficients (do **NOT** solve for the coefficients).

$$y'' - 4y' + 5y = 3\sin t.$$

5. (12) Determine (i) the complementary solution, and (ii) the form of a particular solution using the Method of Undetermined Coefficients (do **NOT** solve for the coefficients).

$$y'' - 6y' + 9y = e^{3t}.$$

6. (12) Use Variation of Parameters to find a particular solution of the following differential equation. The given functions, y_1, y_2 satisfy the corresponding homogeneous equation.

$$2t^2y'' - ty' + y = t\sqrt{t}, \qquad y_1 = \sqrt{t}, \quad y_2 = t.$$

7. (15) Consider the Initial Value Problem:

$$xy'' - (1+x)y' + y = 0, \qquad y(0) = 0, \;\; y'(0) = 0.$$

   (a) (10) $y_1 = e^x$ is a solution to the above equation. Find a second linearly independent solution.

   (b) (5) Solve the IVP; you must find a (non-unique) solution other than $y = 0$.

8. (15) Consider the differential equation:

$$y'' + p(x)y' + q(x)y = 0.$$

   (a) (10) If $y_1 = x$ and $y_2 = x\ln x$ are both solutions to the equation, find $p(x)$ and $q(x)$.

   (b) (5) Do y_1 and y_2 constitute a fundamental set of solutions? Clearly state your reasons.