Math 222 EXAM I, February 9, 2005

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (9) For each differential equation determine (i) its order, and (ii) whether it is linear or nonlinear.

$$(a)\ \ y'' + y = x^2 y, \qquad (b)\ \ \frac{d^2y}{dx^2} + e^x = \frac{dy}{dx}, \qquad (c)\ \ yy''' = \frac{1}{x}.$$

2. (14) Solve the Initial Value Problems (IVP's):

$$(a)\ \ y' + 4y = 8, \qquad y(0) = 4; \qquad\qquad (b)\ \ y' = t\cos(t), \qquad y(\pi/2) = 0.$$

3. (14) Obtain the general solution of the given differential equations:

$$(a)\ \ \frac{d}{dx}\left[\sec(x)\, y\right] = x; \qquad\qquad (b)\ \ 2y'' + y' - y = 0.$$

4. (10) Solve the IVP:

$$xyy' = \ln x, \qquad y(1) = 1.$$

5. (a) (6) Find a differential equation whose general solution is $y = c_1 e^{-t} + c_2 e^{2t}$.
   (b) (6) If the equation in (a) is solved with the initial conditions $y(0) = \alpha$, $y'(0) = 1$, find α so that the solution approaches zero as $t \to \infty$.

6. (10) Solve the IVP:

$$y' = y(1-y), \qquad y(0) = 2.$$

7. (15) Solve the IVP. (Hint: first make the substitution $y = xv$ to transform the given equation into a separable equation for the new variable, v.)

$$2y' = \frac{3y}{x} + \frac{x}{y}, \qquad y(1) = 1.$$

8. (a) (6) Determine the interval on which the IVP is certain to have a unique solution, according to the Existence and Uniqueness Theorem for linear equations. State a reason for your answer.
   (b) (10) Solve the IVP and determine the interval on which the solution exists.

$$xy' + (1-x)y = 2xe^x, \qquad y(1) = e.$$