Math 222 Final Exam, May 11, 2005

1. (a) (4) Is the equation $y'' + ey' = (1 + y)$ linear or nonlinear? What is its order?

   (b) (4) Given $g(t) = \alpha + u_2(t) - 3\alpha u_3(t)$, find α such that $g(1) = 2$.

   (c) (4) Is the function $f(x) = (e^x + e^{-x})\sin(x)$ even, odd or neither? Why?

2. (12) Solve the following IVP's:

$$(a) \ y' + 2y = e^{-2t}, \ y(0) = 1; \qquad (b) \ y'' - 4y' + 20y = 0, \ y(0) = 0, \ y'(0) = 8.$$

3. (12) Solve the following differential equations:

$$(a) \ 2y' - y = \frac{1}{y}, \qquad (b) \ y' + \frac{y}{x} = 1 + y.$$

4. (a) (8) Solve the IVP:

$$\mathbf{X}' = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} \mathbf{X}, \qquad \mathbf{X}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

   (b) (6) Use the Laplace Transform to solve the following IVP:

$$y'' = 2, \quad y(0) = 1, \ y'(0) = 0.$$

5. (14) For each of the following, determine the complementary solution and a particular solution. In (b), you must use the Method of Undetermined Coefficients.

$$(a) \ y'' - 2y' + y = e^t/t, \qquad (b) \ 2y'' - 7y' + 3y = e^t.$$

6. (12) For the following IVP, (a) Solve using Laplace Transforms, (b) evaluate $y(\pi)$, and (c) evaluate $\lim_{t \to \infty} y$.

$$y'' + y = g(t), \quad y(0) = 0, \ y'(0) = 0; \quad \text{where} \ g(t) = \begin{cases} 1, & 0 \le t < 2\pi, \\ 0, & 2\pi \le t, \end{cases}$$

7. (12) One solution to the following equation is $y_1 = x$. Find a second linearly independent solution. Compute the Wronskian of your two solutions.

$$x^3 y'' + xy' - y = 0.$$

8. (a) (4) Carefully sketch the even periodic extension, of period π, of $f(x) = \sin x, \ 0 < x < \pi/2$. Sketch over the interval $x \in [-2\pi, 2\pi]$.

   (b) (8) Find the Fourier Sine series of $f(x) = 1, \ 0 < x < 1$, with period 2.