

Math 222 FINAL EXAM, December 17, 2004

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (16) Solve the Initial Value Problems (IVP's):

$$(a) \quad ty' + y = e^t, \quad y(1) = 0; \quad (b) \quad e^x y' - e^y = 0, \quad y(0) = 0.$$

2. (16) Solve the IVP in (a) and Boundary Value Problem in (b):

$$(a) \quad 2y'' - y' - y = 0, \quad y(0) = 0, \quad y'(0) = 1; \quad (b) \quad y'' = 1, \quad y'(0) = 0, \quad y'(1) = 1.$$

3. (a) (8) Use Undetermined Coefficients to find a particular solution

$$y'' + y' - 2y = 6e^t.$$

- (b) (8) Solve using the Laplace Transform:

$$y'' - y' = \delta(t - 1), \quad y(0) = 0, \quad y'(0) = 0.$$

4. (a) (6) Find the Laplace Transform **using the definition of the Laplace Transform.**

$$f(t) = u_1(t)e^t.$$

- (b) (6) Find the inverse Laplace Transform

$$F(s) = \frac{e^{-4s}}{4s + 1}.$$

- (c) (8) Find the Fourier Series for the function:

$$f(x) = \begin{cases} 1 & -2 \leq x < 0 \\ 0 & 0 \leq x < 2 \end{cases} \quad \text{and} \quad f(x + 4) = f(x).$$

5. (a) (8) Find the general solution in terms of real-valued functions:

$$\mathbf{X}' = \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \mathbf{X}.$$

- (b) (8) Find the eigenvalues and eigenfunctions:

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(\pi) = 0.$$

6. (a) (4) Is the function $f(x) = xe^x - xe^{-x}$ even, odd, or neither? Why?

- (b) (6) Given $g(t) = tu_1(t) - u_3(t)(t - 3)$, determine

$$(i) \quad g(2), \quad \text{and} \quad (ii) \quad \lim_{t \rightarrow \infty} g(t).$$

- (c) (6) Carefully sketch the odd periodic extension of the given function over the interval $[-2\pi, 2\pi]$.

$$f(x) = \cos x, \quad 0 < x < \frac{\pi}{2}.$$