Math 222 EXAM III, November 17, 2004

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (15) A mass weighing 8 lb stretches a spring 2 ft. If the mass is compressed upward 2 ft from its equilibrium position, and given and initial downward velocity of 8ft/s (with no damping) find the position of the mass at any time. Determine the frequency, period, amplitude and phase of the motion. (g=32ft/s^2)

2. (15) Given the matrices **A** and **B** below, compute the following:

   (a) $\mathbf{A}+\mathbf{B}$,

   (b) $\mathbf{AB}$,

   (c) $\mathbf{BA}$.

$$\mathbf{A}=\begin{pmatrix}1 & 0 & 2\\ 3 & 2 & 3\\ 0 & -1 & 3\end{pmatrix}, \qquad \mathbf{B}=\begin{pmatrix}0 & 1 & 2\\ 3 & 1 & 3\\ 1 & -1 & -1\end{pmatrix}.$$

3. (15) Find the Laplace Transform of the given functions **using the definition of the Laplace Transform.**

$$(a)\ \ f(t)=e^{2t}; \qquad (b)\ \ f(t)=\begin{cases} t, & 0\le t<1\\ 2, & 1\le t.\end{cases}$$

4. (15) Find the inverse Laplace Transform of the given functions.

$$(a)\ \ F(s)=\frac{1}{4+s}; \qquad (b)\ \ G(s)=\frac{s+9}{s^2+9}; \qquad (c)\ \ H(s)=\frac{3}{e^{3s}(s-1)}.$$

5. (20) Solve the Initial Value Problems (IVP's) using the Laplace Transform:

$$(a)\ \ 4y''+4y'+y=0, \quad y(0)=0,\ \ y'(0)=-2; \qquad (b)\ \ y''=-\delta(t-2), \quad y(0)=0,\ \ y'(0)=1.$$

6. (20) Consider the initial value problem

$$y''+y=g(t)+\gamma\delta(t-4\pi), \qquad y(0)=0, \quad y'(0)=0; \qquad g(t)=\begin{cases} t, & 0\le t<\pi\\ 2\pi-t, & \pi\le t<2\pi\\ 0, & 2\pi\le t.\end{cases}$$

   (a) (5) Carefully sketch a graph of the forcing function, $g(t)$.

   (b) (10) Find the solution of the given initial value problem.

   (c) (5) Determine the value of γ such that all motion ceases (i.e. $y=0$) for $t\ge 4\pi$. (Useful fact: $\sin(t-\pi)=-\sin(t)$)