Math 222 EXAM II, October 20, 2004

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (15) Find the general solution of the given differential equations:

$$(a)\ 4y'' + y = 0, \qquad (b)\ y''' - y'' = 0, \qquad (c)\ y'' = \frac{1}{2\sqrt{t}}.$$

2. (15) Determine whether the given pair of functions is linearly independent or linearly dependent. Clearly state reasons for your answers.

$$(a)\ f = t,\ \ g = t - 1; \qquad (b)\ f = e^{2t},\ \ g = e^{2(t-1)}; \qquad (c)\ f = \ln t,\ \ g = \ln \frac{1}{t^2}.$$

3. (20) Solve the Initial Value Problems (IVP's):

$$(a)\ y''+2y'+5y = 0, \quad y(\pi/2) = 0,\ \ y'(\pi/2) = 1; \qquad (b)\ 4y''+4y'+y = 0, \quad y(0) = 0,\ \ y'(0) = -2.$$

4. (15) Use the method of Undetermined Coefficients to find a particular solution of the given differential equations.

$$(a)\ y'' - 2y' = 12t^2, \qquad (b)\ y'' + y = \sin(2t) + e^{-t}.$$

5. (15) Find a particular solution of the given differential equation. The given functions, y_1, y_2 satisfy the corresponding homogeneous equation.

$$t^2y'' - ty' + y = t, \qquad y_1 = t, \quad y_2 = t\ln t.$$

6. (20) Consider the Initial Value Problem (IVP):

$$ty'' - (2t+1)y' + (t+1)y = 0, \qquad y(-1) = 0,\ \ y'(-1) = 1.$$

   (a) (5) Determine the longest interval in which the IVP is certain to have a unique twice differentiable solution. Clearly state your reasons.

   (b) (10) Given that $y_1 = e^t$ is a solution to the above equation, find a second linearly independent solution.

   (c) (5) Solve the IVP.