Math 222 EXAM I, September 22, 2004

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (8) For each differential equation determine (i) its order, and (ii) whether it is linear or nonlinear.

$$(a)\ \ y'' + yy' = 4, \qquad (b)\ \ y' = y(1-y^3), \qquad (c)\ \ x^2\frac{d^2y}{dx^2} - \frac{dy}{dx} + \sqrt{x} = 0, \qquad (d)\ \ y''' = \frac{\sin x}{y}.$$

2. (14) Solve the Initial Value Problems (IVP's):

$$(a)\ \ y' - 2y = e^{3t}, \quad y(0) = 3; \qquad (b)\ \ \frac{y'}{y} = \frac{\ln y}{x}, \quad y(1) = e.$$

3. (16) Solve the Initial Value Problems (IVP's):

$$(a)\ \ ty' - y = t^3\sin(\pi t), \quad y(1) = 1; \qquad (b)\ \ y' = -e^{y-x}, \quad y(0) = 0.$$

4. (15) The temperature, T, of an object changes at a rate proportional to the difference between the temperature of the object and the temperature of its surroundings, T_A.
   - (a) (5) Write down a differential equation which describes this situation.
   - (b) (5) Solve the equation, together with the initial condition $T(0) = T_0$.
   - (c) (5) How does your solution behave as $t \to \infty$?

5. (a) (5) Find the general solution of the differential equation: $y'' - y' - 6y = 0$.

   (b) (10) Find the solution of the initial value problem, and sketch the graph of the solution.

$$y'' + y' = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

6. (a) (10) Solve the IVP and determine the interval on which the solution is defined

$$yy' + 2x = -1/2, \quad y(0) = -1.$$

   (b) (6) Without solving the problem, determine the interval on which the IVP is certain to have a unique solution, according to the Existence and Uniqueness Theorem for linear equations.

$$(x+1)y' + (x-2)y = \ln(1-x), \quad y(-2) = 4.$$

7. (a) (6) Determine the values of r for which the differential equation has solutions of the form $y = t^r$ (assume $t > 0$).

$$t^2y'' + 6ty' + 6y = 0.$$

   (b) (10) Find the solution of the IVP, and then determine the constant α so that the solution approaches the value of two (i.e. $y \to 2$) as $x \to \infty$.

$$(x^2+1)y' + xy = \alpha\sqrt{x^2+1}, \quad y(0) = 2.$$