# 2nd Common Exam, Math 213, Nov. 6 2002

1. (15 pts.) Use the chain rule to compute $\partial w/\partial t$ for $w(x,y,z) = xy+yz+xz$ where $x(s,t) = s+t$, $y(s,t) = s-t$ and $z(s,t) = t-s$, simplify, and then evaluate at $(-1,1)$.

2. (20 pts.) (a) Compute $D_{\mathbf{u}}f$ for $f(x,y,z) = x^2y+y^2z+xz^2$ in the direction of $\mathbf{v} = 2\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and evaluate at $(1,1,1)$.
(b) For the surface $z = x^3y - xy^3$, find the equation of the tangent planeand the parameterized equations of the normal line at the point $(-1,1,0)$.
(c) In figure 1 below, sketch $\nabla f$ and $-\nabla f$ at the point $P$. What is $\nabla f$ at the point $Q$.

3. (15 pts.) Find all local maximums, local minimums and saddle points for the surface $f(x,y) = 4xy - x^4 - 2y^4$.

4. (20 pts.) Use Lagrange multipliers to maximize the cross sectional perimeter of a rectangular beam cut from a circular log, under the constraint the $(x,y)$ (the right corner of the beam) lies on the surface of the log. Assume the radius of the log is $\sqrt{2}$ (so that $x^2+y^2=2$). You will need to do the following steps
(a) Write the perimeter of the cross section in terms of $x$ and $y$.
(b) Find the $x$ and $y$ which give the maximum perimeter. What is the maximum perimeter?

5. (15 pts.) Reverse the order of integration and evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1+x^3}\; dxdy.$$

6. (15 pts.) Let $F(x,y,z) = x^2+y^2+z^2-1 = 0$. Use implicit differentiation to evaluate $\left(\frac{\partial x}{\partial y}\right)_z$, $\left(\frac{\partial y}{\partial z}\right)_x$, and $\left(\frac{\partial z}{\partial x}\right)_y$. What is the product $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y$?