CALCULUS 213-FINAL EXAM-DECEMBER 17, 2004

1) Determine the equation of the plane formed by the line $x = 2t - 1$, $y = 2 - t$, $z = 4 - 2t$ and the point (1,-1.-1)

2)Determine using the chain rule, the approximate change in the volume (ΔV) for a cylinder of radius 10 and height 10 if the radius is changed by $\frac{1}{10}$ and the height by $-\frac{1}{10}$ (recall $V = \pi r^2 h$)

3)Find and classify the critical points, for $f(x,y) = x + y + \frac{1}{xy}$

4)Using Lagrange multipliers find the shortest distance between the plane $x + 2y + 4z = 20$ and the origin (0,0,0) (hint:minimize the square of the distance to the origin)

5)Evaluate the line integral $\int_{(1,0,1)}^{(2,1,0)} \mathbf{F} \cdot d\mathbf{R}$ for the conservative vector field

$\mathbf{F} = (y + z^2)\mathbf{i} + (x + 1)\mathbf{j} + (2xz + 1)\mathbf{k}$
by determining the potential function and the change in this potential.

6) For the vector field $\mathbf{F} = -y\mathbf{i} + xy\mathbf{j}$ evaluate the integral $\oint \mathbf{F} \cdot d\mathbf{R} = \oint -ydx + xydy$ around the triangular region enclosed by the curves $y = -2x + 4$, $x = 0$ and $y = 0$ in the first octant.

7) Using Greens Theorem evaluate the integral $\oint \mathbf{F} \cdot d\mathbf{R}$ in problem (6) as a double integral.

8)Using the Divergence Theorem evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, as a volume integral over the region enclosed by the sphere $x^2 + y^2 + z^2 = 4$ for the vector field $\mathbf{F} = x^2\mathbf{i} + xz\mathbf{j} + 3z\mathbf{k}$

9)Use Stokes Theorem to evaluate the circulation $\oint \mathbf{F} \cdot d\mathbf{R}$, as a surface integral, for $\mathbf{F} = 2x\mathbf{i} - 2z\mathbf{j} + y\mathbf{k}$ around the curve which is the boundary of the triangle cut from the plane $x + y + z = 1$, in the first octant, counter clockwise when viewed from above.