

## CALCULUS 211-FINAL EXAM-DECEMBER 15, 2004

1) Determine the equation of the plane formed by the intersecting lines

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{4} = t \text{ and } \frac{1-x}{1} = \frac{2-y}{2} = \frac{z+1}{2} = t$$

2) Evaluate  $\frac{\partial z}{\partial x}$  at the point (2,2,1) for the surface  $x \ln z + xyz^2 + zy = 5$

3) Determine, for  $w = x^2y^3 \sin z + x$  at the point  $(2, 1, \frac{\pi}{6})$

a) Maximum value of the directional derivative  $\frac{dw}{ds}$

b) The equation of the plane tangent to the surface  $x^2y^3 \sin z + x = 4$  at the same point

4) For a point moving along the space curve  $x = t^2 + 1$ ,  $y = \cos t$ ,  $z = e^{2t}$  determine the cosine of the angle between the position and acceleration vectors, at  $t=0$

5) Find the critical points and classify them for  $z = x^4 - 8x^2 + y^2 - 4y$

6) Using Lagrange multipliers, find the point on the line  $y = -2x + 4$  that is closest to point (0,1) (hint: minimize the square of the distance between the points)

7) Evaluate the double integral  $\int_0^1 \int_{\sqrt{x}}^1 \left(\frac{1}{\sqrt{1+y^3}}\right) dy dx$  by reversing the order of integration

8) Evaluate the volume in the region bounded by the parabolic cylinder  $y = x^2$  and the planes  $y + z = 1$  and  $z = 0$ , by evaluating the volume integral  $\left(\iiint dV\right)$

9) a) Determine the potential of the conservative vector field  $\mathbf{F} = xz^2\mathbf{i} + 2y\mathbf{j} + x^2z\mathbf{k}$

b) Evaluate the work done by this vector field in moving along an object from the point (0,0,1) to (1,2,1)

10) For the space curve  $x = t^2 + 1$ ,  $y = \frac{t^4}{4}$ ,  $z = t^3 - 1$  evaluate the line integral  $\int_c \mathbf{F} \cdot d\mathbf{R}$  for  $\mathbf{F} = 5y\mathbf{i} + 7z\mathbf{j} + x\mathbf{k}$  between  $0 \leq t \leq 1$

11) For the vector field  $\mathbf{F} = -y\mathbf{i} + xy\mathbf{j}$  evaluate as a line integral  $\oint \mathbf{F} \cdot d\mathbf{R} = \oint -ydx + xydy$

around the region enclosed by the curves  $y = 4x$  and  $y = x^3$  in the first octant.

12) Using Greens Theorem evaluate the integral  $\oint \mathbf{F} \cdot d\mathbf{R}$  in problem (11) by double integration.