Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (9 points) Differentiate:

$$(a)\ 2\arcsin(\sqrt{x}) - \arcsin(2x-1), \qquad (b)\ \ln\left(\frac{2+3x^2}{4-x}\right), \qquad (c)\ \log_2(x) + \cosh(3x).$$

2. (9 points) Compute the following limits:

$$(a)\ \lim_{x\to\infty} \frac{3x^2-2x+1}{2x^2-x+2}, \qquad (b)\ \lim_{\theta\to 0} \frac{\sin 2\theta}{\theta + 3\tan\theta}, \qquad (c)\ \lim_{x\to 0^+} x^x$$

3. (12 points) Integrate:

$$(a)\ \int \frac{2+3x}{1+9x^2}\,dx, \qquad (b)\ \int x\ln(2x)\,dx, \qquad (c)\ \int \tan^4 x\,dx.$$

4. (12 points) Integrate (evaluate the improper integrals *correctly*):

$$(a)\ \int \frac{x}{(x+3)(x+4)}\,dx, \qquad (b)\ \int \frac{1}{\sqrt{5-4x-x^2}}\,dx, \qquad (c)\ \int_{-1}^{2} \frac{1}{\sqrt{2-x}}\,dx.$$

5. (8 points) Find the first three non-zero terms of the Taylor series for $f(x) = \sqrt{1+4x}$ at $a = 2$.

6. (9 points) Determine whether the following series converge or diverge. Find the sum of those that converge. Justify your answer!

$$(a)\ \sum_{n=0}^{\infty} \left(2\frac{(-1)^n}{4^n} + \frac{4}{2^n}\right), \qquad (b)\ \sum_{n=0}^{\infty} \cos(\pi n), \qquad (c)\ \sum_{n=0}^{\infty} \frac{1}{n!}.$$

7. (12 points) Determine whether the following positive term series converge or diverge. State clearly which test you use.

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{n^2+2n+3}, \qquad (b)\ \sum_{n=1}^{\infty} \left(\frac{2n}{3n+1}\right)^n, \qquad (c)\ \sum_{n=2}^{\infty} \frac{2}{n\ln n}.$$

8. (12 points) Determine whether the following series converge absolutely, converge conditionally, or diverge. Justify your answer!

$$(a)\ \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^3+1}}, \qquad (b)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}, \qquad (c)\ \sum_{n=1}^{\infty} \frac{\sin n}{n^3}.$$

9. (9 points) Let $r = 4\sin\theta$ define a curve in polar coordinates.
   (a) Sketch this curve.
   (b) Compute the area of the region in the plane enclosed by this curve.
   (c) Write down an equation for this curve in cartesian coordinates. Interpret it geometrically.

10. (8 points) For the power series below, find (i) the radius of convergence, and (ii) the interval of convergence (including both endpoints).

$$\sum_{n=1}^{\infty} \frac{x^n}{4^n(2n+1)}.$$