Math 112 **Exam 3** April 13, 2005

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (18 points) Compute the following integrals (do it **correctly** for improper integrals):

$$(a)\ \int \frac{\cos x}{\sin^2 x - 3\sin x + 2}\,dx, \quad (b)\ \int_{-2}^{0} \frac{1}{(x+2)^{1/3}}\,dx, \quad (c)\ \int_{-\infty}^{2} xe^{-x^2}dx.$$

2. (18 points) Determine whether the following sequences converge, and if so, find their limits

$$(a)\ a_n = \frac{3n^2+2n+1}{4n^2+5n+6}, \quad (b)\ 1,2,1,2,1\ldots, \quad (c)\ a_n = \left(1+\frac{2}{n}\right)^n.$$

3. (16 points) Let $f(x) = e^{-2x}$.

   (a) Find the Maclaurin series of $f(x)$.

   (b) Write down the 3rd-degree Taylor Polynomial of $f(x)$ centered at $x = 0$.

   (c) Use this Taylor Polynomial to give an estimate for the number e.

4. (18 points) Determine whether the following series converge or diverge, and, if they do, compute their value (*Hint:* in c, show that the series is telescoping)

$$(a)\ \sum_{n=0}^{\infty} \frac{n}{3n+1}, \quad (b)\ \sum_{n=0}^{\infty} \frac{2\cdot 3^n - 1}{4^n}, \quad (c)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}.$$

5. (18 points) Use Comparison Tests (in a,c) or Integral Test (in b,c) to determine whether the following series converge or diverge

$$(a)\ \sum_{n=3}^{\infty} \frac{1}{n-2}, \quad (b)\ \sum_{n=1}^{\infty} \frac{2n}{(n^2+4)^2}, \quad (c)\ \sum_{n=1}^{\infty} \frac{\sqrt{n}\ln n}{n^2+1}.$$

6. (12 points)

   (a) Find the rational number represented by $0.33333\ldots$.

   (b) Find all possible values of r for which

$$\sum_{n=0}^{\infty} r^n = -\frac{9r}{4}.$$