

Read each problem carefully. Please show all your work for each problem!
Use only those methods discussed thus far in class. No calculators!

1. (15 points) Differentiate:

$$(a) f(x) = e^{2x} \sin 3x, \quad (b) g(y) = \frac{\ln(y^2)}{y}, \quad (c) h(z) = \log_x(2x).$$

2. (15 points) Integrate:

$$(a) \int x \cdot 3^{x^2} dx, \quad (b) \int \frac{\log_4 x}{x} dx, \quad (c) \int \frac{e^{2x}}{3 + e^{2x}} dx.$$

3. (15 points) Evaluate the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}, \quad (b) \lim_{x \rightarrow \infty} (x^2 + 2x + 3)e^{-x}, \quad (c) \lim_{x \rightarrow 0} \frac{1 - \cos x}{\ln(1 - x^2)}.$$

4. (15 points) Evaluate the following limits:

$$(a) \lim_{x \rightarrow 0^+} x^{\sqrt{x}}, \quad (b) \lim_{z \rightarrow \infty} \left(1 + \frac{1}{2z}\right)^z, \quad (c) \lim_{t \rightarrow \infty} \frac{\ln t}{t^{0.01}}.$$

5. (14 points) Find the length of a curve

$$y = \frac{x^2}{2} - \frac{\ln x}{4}, \quad 2 \leq x \leq 4.$$

6. (12 points) Find the area of the surface of revolution obtained by revolving the curve

$$y = \left(x - \frac{4}{9}\right)^{3/2}, \quad 1 \leq x \leq 4,$$

around the y -axis.

7. (14 points) How much work is needed to fill a spherical tank of radius 20 ft with its center 30 ft above the ground, with water ($\rho = 63 \text{ lb/ft}^3$) from the ground level?