

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (10 points) Compute:

$$(a) \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2\theta^2}, \quad (b) \lim_{x \rightarrow \infty} x^2 e^{-2x}, \quad (c) \lim_{x \rightarrow 0^+} (1 + 2x)^{1/x}.$$

2. (10 points) Integrate:

$$(a) \int_0^2 \frac{dx}{\sqrt{16 - x^2}}, \quad (b) \int x \cos 2x \, dx, \quad (c) \int \sin^3 x \, dx$$

3. (12 points) Integrate (evaluate the improper integrals *correctly*):

$$(a) \int \frac{dx}{x^2 - 3x - 4}, \quad (b) \int \frac{2x + 1}{x^2 + 4} \, dx, \quad (c) \int_1^{\infty} \frac{dx}{x^2}.$$

4. (12 points) Find the area of the region in polar coordinates lying outside the curve $r = 1$ and inside $r = 2 \cos \theta$.

5. (10 points) Find the first two non-zero terms in the Maclaurin Series of the function $f(x) = \arctan x$.

6. (10 points) Determine whether the following series converge or diverge. Find the sum of those that converge. Justify your answer!

$$(a) \sum_{n=1}^{\infty} n, \quad (b) \sum_{n=0}^{\infty} \frac{2^n - 3^n}{4^n}, \quad (c) \sum_{n=0}^{\infty} \frac{1}{x^n}.$$

7. (12 points) Determine whether the following positive term series converge or diverge. State clearly which test you use.

$$(a) \sum_{n=0}^{\infty} \frac{n}{n^4 + 1}, \quad (b) \sum_{n=1}^{\infty} \frac{2^n}{n!}, \quad (c) \sum_{n=1}^{\infty} \frac{\ln n}{n}$$

8. (12 points) Determine whether the following series converge absolutely, converge conditionally, or diverge. Justify your answer!

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}, \quad (b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+2}}, \quad (c) \sum_{n=0}^{\infty} e^{-n} \sin n.$$

9. (12 points) For the power series below, find (i) the radius of convergence, and (ii) the interval of convergence (including both endpoints).

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n x^n}{n}.$$