Math 112 **Exam 3** November 17, 2004

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (12 points) Determine whether or not the following improper integrals converge. Evaluate the ones that do converge.

$$(a)\ \int_{-\infty}^{+\infty} \frac{dx}{x^2+1}, \qquad (b)\ \int_0^4 \frac{dx}{x\sqrt{4-x}}.$$

2. (12 points) Determine if the following sequences converge, and if so, find their limits

$$(a)\ 1, 2, 3, 4, \ldots \qquad (b)\ a_n = \frac{1-5n^4}{n^4+7n^3}, \qquad (c)\ a_n = -1 + \left(\frac{1}{11}\right)^n.$$

3. (10 points) Find the rational number represented by $1.171717171717\ldots$

4. (12 points) Determine whether the following series converge, and if they do, compute their sums

$$(a)\ \sum_{n=1}^{\infty} \frac{3^n - 2^{2n}}{5^n}, \qquad (b)\ \sum_{n=1}^{\infty} \ln\left(\frac{2n-1}{2n+1}\right).$$

5. (12 points) Find the 2-nd degree Taylor polynomial for the function $f(x) = \sqrt{3+x}$ with $a = 1$. Use it to approximate $\sqrt{5}$.

6. (10 points) Find the Maclaurin series of $f(x) = \cos 3x$.

7. (12 points) Use the Integral Test to verify convergence or divergence of the following series

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{(n+3)^3}, \qquad (b)\ \sum_{n=5}^{\infty} \frac{1}{n(\ln n)^5}.$$

8. (10 points) Use Comparison Tests to determine whether the following series converge or diverge

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{(n+3)^3}, \qquad (b)\ \sum_{n=2}^{\infty} \frac{\cos n}{n^2-1}.$$

9. (10 points) Determine whether the following series converge absolutely, converge conditionally, or diverge

$$(a)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^5+9}}, \qquad (b)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{9n+1}}.$$