

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (12 points) Evaluate the following limits:

$$(a) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right), \quad (b) \lim_{x \rightarrow +\infty} \frac{x^2}{\sqrt{1+9x^4}}, \quad (c) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x}.$$

2. (12 points) Find dy/dx :

$$(a) y = 3^x + 1, \quad (b) y = x \ln(x^2 + 1), \quad (c) y = x^{x^2}.$$

3. (12 points) Integrate the following:

$$(a) \int 2^{3x} dx, \quad (b) \int \frac{(\ln 2x)^2}{x} dx, \quad (c) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$$

4. (12 points) Evaluate the following limits:

$$(a) \lim_{t \rightarrow +\infty} \frac{\ln t}{t^2}, \quad (b) \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{2x} \right)^x, \quad (c) \lim_{x \rightarrow 0^+} (\sin x)^x.$$

5. (12 points) Solve each of the following for z :

$$(a) 3^{-z} = 81, \quad (b) \ln z^2 + \ln z^{-3} = 1, \quad (c) e^{2z} - e^z - 2 = 0.$$

6. (10 points) A spring of natural length 1 m needs to be stretched another 0.25 m to hold the force of 1000 N. How much work needs to be done to compress the spring from its natural length to 0.7 m?

7. (10 points) Find the length of a curve

$$y = -\frac{x^4}{4} - \frac{1}{8x^2},$$

from $x = \frac{1}{2}$ to $x = 1$.

8. (10 points) Find the area of the surface generated by revolving the curve $y = \sqrt{4x - x^2}$, with $2 \leq x \leq 3$, around the x -axis.

9. (10 points) Compute:

$$(a) \int \frac{dx}{1 + e^{-x}}, \quad (b) \lim_{x \rightarrow a} \left(\frac{\sin x}{\sin a} \right)^{\frac{1}{x-a}}.$$