

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (18 points) Compute

$$(a) \int (4x^3 - 3x^2 + 1) dx, \quad (b) \int \frac{\sqrt{y} + 3}{\sqrt{y}} dy, \quad (c) \int (2 \sec^2 z - z^2) dz$$

2. (16 points) Compute dy/dx and d^2y/dx^2 for the following functions

$$(a) \cos y = x, \quad (b) y = \int_1^x \tan(\theta^2) d\theta.$$

3. (12 points) Solve the following Initial Value Problem:

$$\frac{dy}{dx} = 1 - \sqrt{x+1}, \quad y(0) = 1.$$

For the solution $y(x)$, find $\lim_{x \rightarrow +\infty} y(x)$.

4. (18 points)

- (a) Evaluate

$$(a) \int_{-\pi/2}^{\pi/6} \sin 2z dz, \quad (b) \int_{-2}^3 |x+1| dx.$$

- (b) Find all the horizontal and vertical asymptotes of

$$f(x) = \frac{x}{x+2}.$$

5. (18 points) Let $f(x) = -x^3 + 2x^2 + 4x - 1$.

- (a) Use the First Derivative Test to find all the open intervals on which $f(x)$ is increasing or decreasing.
(b) Use the Second Derivative Test to find all the open intervals on which $f(x)$ is concave up or down.
(c) Sketch the graph of $f(x)$ indicating local extrema, inflection points, and the behavior at infinity.

6. (18 points)

- (a) Use the summation notation to write

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}}.$$

- (b) Write down the Riemann sum for the integral

$$\int_0^3 \frac{1}{\sqrt{2x+1}} dx,$$

using regular partition of the given interval into 4 subintervals and $x_i^* = x_{i-1}$, the left endpoint of each subinterval.

- (c) Compute the value of the integral. What is the average value \bar{y} of $y = \frac{1}{\sqrt{2x+1}}$ over the interval $[0, 3]$?