Math 111 **Final Exam** December 15, 2004

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (9 points) Differentiate the following functions:

$$(a)\ f(x) = (2x+3)(1-x), \qquad (b)\ g(y) = \frac{y}{(1+y^2)^{1/2}}, \qquad (c)\ h(\theta) = \tan(2\theta) + 5\cos(\theta^2).$$

2. (9 points) Compute the following limits:

$$(a)\ \lim_{x\to\infty} \frac{2+3x-x^2}{1+2x+3x^2}, \qquad (b)\ \lim_{t\to 1} \frac{t-1}{2-\sqrt{5-t}}, \qquad (c)\ \lim_{z\to 0} \frac{\sin 3z}{2z}.$$

3. (8 points) Use the definition of derivative to calculate $f'(x)$, if $f(x) = \frac{1}{1+3x}$.

4. (8 points) Write down the linear approximation $L(x)$ for the function $f(x) = \sqrt{3+x}$ near the point $a = 1$. Use this linear approximation $L(x)$ to estimate $\sqrt{5}$.

5. (8 points) Show that the function $f(x) = \frac{1}{x}$ satisfies the assumptions of the Mean Value Theorem on the interval $[2,3]$. Find all numbers c in this interval that satisfy the conclusions of that Theorem.

6. (15 points) For the function

$$f(x) = \frac{x}{x^2+4},$$

find the following, if they exist: (i) all local extrema, (ii) intervals where the function increases or decreases, (iii) all points of inflection, (iv) intervals of upward or downward concavity, (v) all asymptotes. Also, sketch a plot of the curve $y = f(x)$.

7. (8 points) Find the area of the region in the plane bounded by the curves $y = -x^2$ and $y = x^2 - 2x - 4$.

8. (9 points) Integrate:

$$(a)\ \int \left(x^{3/2} - \frac{2}{x^{1/2}}\right) dx, \qquad (b)\ \int 3t\cos(t^2)dt, \qquad (c)\ \int_0^{\pi/4} \tan x \sec^2 x\, dx$$

9. (9 points) Find two positive numbers x and y, such that $3x+4y = 5$, for which the expression $x^2 + y^2$ is as small as possible.

10. (9 points) Find the volume of the solid generated by rotating the plane region bounded by the curves $y = 2x^2$, $y = 3x^2$, $y = 1$, $y = 4$, around the y-axis.

11. (8 points) Let x_i^* denote a selected point in the i-th subinterval $[x_{i-1}, x_i]$ of the partition of the interval $[-1,2]$ into n subintervals each of length Δx. Calculate the following limit by identifying the expression below with a Riemann sum of the appropriate integral and calculating that integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^*(3x_i^* + 2)\Delta x.$$