Math 111 **Exam 3** November 17, 2004

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (12 points) Compute

$$(a)\ \int \sqrt{x+2}\,dx, \qquad (b)\ \int \frac{2t^3+1}{t^2}\,dt, \qquad (c)\ \int (3\sin 2z + 4z)\,dz$$

2. (12 points) Show that of all rectangles of area 25 the square has the minimum perimeter. Use the First Derivative Test to verify your answer.

3. (12 points) Find all the open intervals on which the function $y = -2x^3 + 6x^2 - 3$ is increasing or decreasing. Sketch the graph of this function indicating the local extrema and the behavior at infinity.

4. (12 points) Find all the asymptotes of the function

$$f(x) = \frac{2x-3}{7x+4}.$$

5. (14 points) Sketch the graph of the function $y = x(x-5)^{2/3}$. Find all the critical points, inflection points, and the correct concave structure. Apply the second derivative test at each critical point, if possible.

6. (12 points) Compute

$$(a)\ \int_3^5 (x+1)\,dx, \qquad (b)\ \int_{-2}^{2} (2-|x|)\,dx.$$

7. (14 points)

   (a) Use the summation notation to write

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}.$$

   (b) Write down the Riemann sum for the integral

$$\int_3^5 (x+1)\,dx,$$

   using regular partition of the given interval into n subintervals and $x_i^* = x_{i-1}$, the left endpoint of each subinterval.

   (c) Calculate the integral as a limit of Riemann Sums and compare it with the value obtained in Problem 6(a). *Hint:* $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

8. (12 points) For $f(x) = x^2 - 2x$, find the point $\bar{x}$ on the interval $[-2, 1]$ at which $\bar{y} = f(\bar{x})$, where $\bar{y}$ is the average value of $y = f(x)$.