Math 111 **Exam 2** October 20, 2004

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. No calculators!

1. (16 points) Differentiate:

$$(a)\ \ f(t) = t^2\cos(t^3), \qquad (b)\ \ g(y) = \cos(3\tan(2y)),$$
$$(c)\ \ h(z) = \frac{\sin z}{z}, \qquad (d)\ \ u(x) = \frac{1}{(5-2\sqrt{1+x})^{1/3}}.$$

2. (12 points) Use linear approximation of the function $f(x) = \sqrt{x}$ around $x = 100$ to approximate the value of $\sqrt{121}$. How does it compare with the exact value?

3. (15 points) Find the absolute minimum and the absolute maximum of the following functions, if they exist

$$(a)\ \ f(x) = -x + 1,\ \ [-4,-1], \qquad (b)\ \ g(x) = \frac{x}{x^2+2},\ \ [-1,2],$$
$$(c)\ \ h(x) = 2 - |x|,\ \ [-3,1].$$

4. (14 points) Use implicit differentiation to find the equation of the tangent line to the curve $y^2 - 2x - 4y - 1 = 0$, passing through the point $(-2, 1)$. Verify that this point lies on the curve.

5. (14 points) A blimp is hovering 80 meters above the ground. A car is moving with speed 20 m/sec away from the point directly under the blimp. At what rate is the distance between the blimp and the car increasing when the car is 60 m away from the point directly under the blimp?

6. (16 points) What is the price of the cheapest cylindrical can that can be made of copper sheet costing 2 cents/cm^2, if it is to hold 54π (≈ 170) cm^3 of contents? *Hint:* minimize the total surface area of the can of the given volume.

7. (13 points) Use Newton's method to find successive approximations to the root of the function $f(x) = x^{-1} - 2$ and compare it with the exact value of the root.

   (a) Use $x_0 = \frac{1}{2}$ as the initial guess and perform two iterations of the Newton's method. Do the iterations converge? To the exact root?

   (b) Now use $x_0 = 0$ as the initial guess and perform two iterations of the Newton's method. Do the iterations converge? To the exact root?

   (c) Repeat with $x_0 = -1$ as the initial guess and perform three iterations of the Newton's method. Do the iterations converge?