

Proposal for a New Graduate Course

Course Proposed by: Michael Booty, Mathematical Sciences Department

Date Proposed: March 9, 2006

Departmental Approval and Date:

Dean's Approval and Date:

Date to be Presented to the Graduate Council: 03/23/06

1. Catalog Description

A. Course Number, Title, Credits

Math 639, Mathematical Modeling II, 3 credits.

B. Prerequisites

Math 613 Advanced Applied Mathematics I, Modeling, or equivalent.

C. Proposed Catalog Description

Continuation of Math 613 (Advanced Applied Mathematics I, Modeling). Concepts and strategies of mathematical modeling are developed by investigation of case studies in a selection of areas. Topics are complementary to those of Math 613 and include, for example, the mathematical theory of elasticity and electromagnetism.

D. When will the course first be offered?

Spring 2007.

2. Purpose of the Course

A. Why is the course needed?

Applied mathematicians in industry and academia are expected to have a working familiarity with an increasingly wide range of applications. This course is intended to develop mathematical modeling skills in areas that complement those presented in Math 613, and which are likely to complement the area of application in the thesis topic of most doctoral candidates in the Mathematical Sciences Department. It is intended to add breadth to the modeling skills and familiarity with applications of advanced graduate students in Applied Mathematics.

The choice of topics identified (elasticity and electromagnetics) is motivated by their being studied classically in the applied mathematics curriculum and by their continuing relevance to developments in modern technology (such as micro-devices, biological applications etc.).

The topics presented in Math 613 are usually: traffic flow (as an introduction to continua), fundamentals of fluid dynamics, and various examples of wave propagation. One of these topics is sometimes substituted with models of population dynamics or mechanics.

The proposed course will emphasize the similarities (and differences) in the mathematical formulation and treatment between the different topics studied.

B. For whom is the course intended?

The course is intended for Mathematical Sciences Ph.D. students and for Applied Mathematics Masters students, who may take the course as an elective. Students in other programs may choose to take the course as an elective, if desired.

The course is not intended as a substitute for dedicated one or two-semester courses offered by other departments, such as Classical Electrodynamics I (Phys 621) and II (Phys 721), or Theory of Deformable Solids in Mechanical Engineering I (ME 785) and II (ME 786). The Mathematical Sciences Department strongly encourages its PhD students to take courses offered by other departments, such as Physics and Mechanical Engineering, when it is appropriate to their research interests.

C. Reason for prerequisites

The proposed course forms a continuation of and complement to Math 613 (the prerequisite). In combination, the two courses provide a foundation in the mathematical treatment of topics that are usually studied in the Applied Mathematics curriculum (i.e., Mechanics, Fluid Dynamics, Solid Mechanics, and Electromagnetics).

3. Course Details

A. Course Outline. Attach outline including topics by week, examinations including the final examination, for a 15 week course; attach any additional information that is appropriate.

The proposed course consists of two areas of mathematical modeling, elasticity and electromagnetics. Please see Item 5 below for a detailed breakdown of the topics by week.

B. Textbook and References

There are many texts covering the material of this course. Texts appropriate for the outline that follows, are

1. "Mathematics Applied to Continuum Mechanics" by L.A. Segel, Dover, NY (1987). ISBN 0-486-65369-2. (To be reissued in the SIAM Classics in Applied Mathematics series.)
2. "Theory of Elasticity" Third edition, by L.D. Landau and E.M. Lifshitz, Elsevier Butterworth Heinemann (1986). ISBN 0-7506-2633-X.

3. "Linear Elastic Waves" by J.G. Harris, Cambridge University Press, (2001). ISBN 0-521-64368-6.
4. "A First Course in Continuum Mechanics" Third edition, by Y.C. Fung, Prentice Hall (1994). ISBN 0-13-061524-2.
5. "Classical Electrodynamics" Third edition, by J.D. Jackson, Wiley (1999). ISBN 0-47-130932-X.
6. "Electrodynamics of Continuous Media" Second edition, by L.D. Landau and E.M. Lifschitz, Elsevier Butterworth Heinemann (1984). ISBN 0-75-062634-8.
7. "Wave Motion" by J. Billingham and A.C. King, Cambridge University Press (2000). ISBN 0-521-63257-9.
8. "The Theory of Electromagnetism" by D.S. Jones, Macmillan (1964).
9. "Electromagnetic Theory" by J.A. Stratton, McGraw-Hill (1941). ISBN 0-07-062150-0.

(The last two references are old but highly-regarded classics on reserve in the library. There are more recent texts that cover the material well and may also be appropriate but sometimes lack the combination of insight and simplicity in treatment of canonical problems offered by older sources.)

C. Number of hours of lecture, recitation, and laboratory

Three hours of lectures per week, no recitation or laboratory.

4. Course Mechanics

A. Who can and will teach the course? How will the addition of this course impact current faculty loading?

This course can be taught by about ten faculty in the Department of Mathematical Sciences (Kriegsmann, Stickler, Luke, Papageorgiou, Siegel, Booty, Bechtold, Moore, Tao, and Young). The course has been offered once previously under the Math 707 Special Topics heading. Its addition will not adversely affect current faculty loading.

B. How often will the course be offered? Will any existing course be deleted from the catalog in favor of this course?

Once every two or three years, in the Spring semester as a sequel to Math 613. No existing course will be deleted in favor of this course.

C. What cost is associated with introducing the course?

No costs are associated with introducing the proposed course.

D. What space is required?

A standard size classroom. Adequate space and facilities already exist on campus.

E. What new equipment is needed? What existing equipment is to be used? (Include computer and library needs)

No new equipment is needed. Existing NJIT and Department of Mathematical Sciences computing resources (including Matlab etc.) and Library facilities are sufficient to complete anticipated assignments.

F. What staff time is required? (Consider faculty, support staff, and graduate assistants.)

The usual faculty time required for the teaching of a graduate level course, e.g., 3 hours teaching, 2 hours preparation, and 2 hours grading each week.

G. Estimated enrollment

From ten to fifteen students.

H. How will student performance and grades be determined?

Homework assignments 60%, Midterm 15%, and Final exam 25%, or similar as determined by the instructor.

I. For whom is the course designed? At what graduate level will the course be offered: Masters, PhD, or an intermediate level? What degree program or programs will it apply to?

The course is designed for second and third-year graduate students in the Mathematical Sciences (Applied Mathematics) PhD program and for final semester students in the MS Applied Mathematics programs (as an elective). It may also be suitable for students from other departments, as advised by their departments' advisors. The models studied will be drawn from applications.

J. Will the course be offered by any non-traditional methods

No. Except that the course may be team-taught, as determined by faculty in consultation with the department chair.

K. Describe any unusual features of the course

There are no unusual features of the course.

L. Will the course be evaluated in any way?

The course will be subject to evaluation by the usual means of course evaluations completed by students in the class, and the usual means of program assessment and review.

5. Course outline (15 week schedule)

Weeks

- 1 Strain tensor, linearization, and decomposition of arbitrary infinitesimal deformation. Compatibility of strain and displacement. Stress tensor and the linear constitutive relation (Hooke's law).
- 2 Conservation of mass, momentum, and energy. Linearization for small deformation, Navier's equation. Boundary conditions. Elastic (or strain) energy. Assignment 1.
- 3 The principal of virtual work. Basic uniqueness results of static and dynamic elasticity. Potential energy minimization in equilibrium. Reciprocity results (Theorem of Betti and Rayleigh).
- 4 Problems of static deformation. Bending and torsion (St Venant problem) of a rod. Airy stress function. Use of reciprocity results.

- 5 Vibration of rods and plates. Longitudinal and transverse vibrations. Dynamic model for deformation of a long thin beam ('anomalous' dispersion). Assignment 2.
- 6 Elastic waves in an unbounded domain, plane wave solutions. Dilatational (P) waves and rotational (S) waves. Reflection of a plane (SV) wave at an interface. Rayleigh (surface) waves.
- 7 Internal reflection. Love (surface SH) waves in a layer over a substrate. Midterm.
- 8 Maxwell's equations in a macroscopic medium. Laws of Gauss, Ampere, and Faraday. Charge and current density as sources or inhomogeneities in Maxwell's equations. Definitions of (E,B) and (D,H). Conservation of charge. Constitutive relations: polarization and magnetization in macroscopic media, Ohm's law, linear isotropic media. Lorentz force.
- 9 Derivation of boundary conditions at an interface (including surface charge and surface current). Electrostatics and Magnetostatics. Examples, potentials, Biot-Savart law. Assignment 3.
- 10 Electrostatic and Magnetostatic energy density. Conservation of energy and Poynting's theorem. The differential form of the conservation law. Conservation of momentum and Maxwell's stress tensor.
- 11 Plane, unbounded electromagnetic waves (i) in a non-conducting (lossless) medium, and (ii) in a conducting (lossy) medium. Polarization. Assignment 4.
- 12 Reflection and refraction at a plane interface between dielectric media.
- 13 Waveguides: TE, TM, and TEM modes; cut-off frequency.
- 14 Radiation from a bounded, time-dependent current distribution. Potentials, Multipole expansion, Far-field expansion.
- 15 Final Exam.