Mathematical Analysis of Applied Loads on Skeletal Muscles in Osteopathic Manual Treatment

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Abstract

Context: To determine the loads to produce compression, shear, extension and twist on biceps muscle in osteopathic manual treatment.

Methods: Mathematical Analysis valid for the in vivo state of transversely isotropic biceps muscle is performed to determine the loads under elastic and viscoelastic biceps models.

Results: 7% lower loads are needed to produce 10% deformation using the viscoelastic model compared to the elastic model. Using the viscoelastic model, it is found that stress relaxes by 18% of its maximum value for the case in which muscle is deformed by 10% over a period of 60 seconds and held in that deformed state up to 200 seconds. It is observed that with quick maneuvers, the viscoelasticity effect is decreased, i.e. greater loads need to be applied for a given deformation.

Conclusions: The biceps muscle is 15 times stiffer in the directions parallel to the muscle fibers compared to the perpendicular direction. The results in this paper can be useful to the manual therapists to adjust their technique to tissue properties. Since the biceps muscle is viscoelastic, the results obtained in this paper for the viscoelastic model are more realistic for determination of viscoelastic stresses compared to those from using the elastic model.

Key Words: Biceps Muscle, Viscoelasticity, Compression, Shear, Extension , Twist, Moment.

Descriptive Summary

A three dimensional mathematical model is developed to establish the relationship between the applied loads and the resulting deformations produced in compression, shear extension, and twist on biceps muscle.

The results indicate that with quick maneuvers, the viscoelasticity effect is decreased, i.e. 50% greater load needs to be applied to obtain the same deformation using a quick maneuver as opposed to a slow maneuver. The biceps muscle is 15 times stiffer in the directions parallel to the muscle fibers compared to the perpendicular direction. 80% of the maximal stress is produced during the first 46 seconds of 60 seconds duration of manual therapy.

Some questions for the continuing medical education quiz.

Question: In the first 46 seconds of the constant strain rate on the muscle,

A. 60% of the maximum stress occurs.

B. 100% of the maximum stress occurs

C. 80% of the maximum stress occurs

Answer: C

Question: For high velocity short duration maneuvers (0.25 seconds), force applied is what percent of that with long duration of 60 seconds.

A. 75 %

B. 150 %

C. 200 %

Answer: B

Question: Has any three dimensional mathematical model been developed prior to that presented in this paper that establishes the relationship between the applied loads and the resulting deformations produced in compression, shear, extension, and twist on biceps muscle?

A. Yes

B. No

C. Not known

Answer: B

1. Introduction

Several forms of manual therapies have been developed which aim to improve postural alignment and other expressions of musculoskeletal dynamics [1-3]. However, excessive loading and high velocity maneuvers may carry some risk and should be approached with caution [3]. At present, mechanical forces are applied intuitively, which can be either helpful or harmful. Therefore, accurate values of mechanically applied forces in osteopathic manual treatment must be scientifically determined. These should be based upon scientifically established in vivo mechanical properties of soft tissues such as skeletal muscles and fasciae, so that manual methods of any kind are helpful and not counterproductive.

Therefore, the in vivo mechanical properties of human skeletal muscles must be known a priori. Recently MR elastography (MRE) has been developed to quantify the mechanical properties of soft biological tissues [4,5]. The shear wave patterns observed in this technique are used to non-invasively evaluate anisotropic elastic coefficients of skeletal muscles [6-8].

The skeletal muscle is considered to be transversely isotropic material for analyzing experimental shear wave speeds [7]. Therefore, assuming biceps muscle as transversely isotropic material, the mechanical properties of biceps muscle were evaluated quantitatively [9]. In their experiment [9], the actuator was applied at the distal tendon of the biceps muscle to induce shear vibration along the principal axis of symmetry (parallel to the muscle fibers direction). The resulting two dimensional waveform profiles of the wave images were used to determine directly the shear modulus of the biceps muscle.

The linear theory of transverse isotropy with small deformation strain tensor [10] was then employed to determine the Young's moduli along the axis of symmetry and along axes in the plane perpendicular to this axis and also the shear moduli along these axes.

In this paper, this quantitative information about the anisotropic elastic properties obtained in [9] has been exploited to determine the elastic and viscoelastic stresses produced in the transversely isotropic biceps muscle when it is subjected to compression, shear, longitudinal extension and twist in manual therapy. For this purpose, the linear stress-strain relation for transversely isotropic material [10] is applied to the transversely isotropic biceps muscle, since we are considering small deformations. Then, the appropriate mechanical forces needed for producing a specific non damaging deformation in the biceps muscle are determined. This can provide useful information to the biophysicists and manual therapy practitioners for beneficial results to their patients.

2. Method

(i) Deformation:

We assume the displacements u_1, u_2, u_3 along X_1, X_2, X_3 produced on the surface muscle element covering its entire thickness, by the manual therapy technique of shear and elongation along the X_3 axis, compression along the negative X_1 axis, the twist about X_3 axis, (Fig.1), to be given by

$$\begin{aligned} u_1 &= (k_3 - 1)X_1 + X_1(Cos\,\psi - 1) - X_2 Sin\,\psi \\ u_2 &= (k_2 - 1)X_2 + X_2(Cos\,\psi - 1) + X_1 Sin\,\psi \\ u_3 &= k_1 X_1 + k_4 X_3 \end{aligned} \qquad (1)$$

where the X_i axes are in the Cartesian coordinate system. Here, X_3 is taken parallel to the muscle fibers direction. k_1 (>0) denotes the shear ratio due to the application of the tangential force. The maximum shear occurs at the surface of the muscle where thickness is maximum, and is zero at the bottom of the muscle where thickness is zero; k_4 (>0) is the extension ratio due to the applied longitudinal force. k_3 (<1) denotes the compression ratio due to the applied normal pressure. k_2 (>0) is the extension ratio, resulting from the compression and shear on the surface of the muscle. ψ is the angle (in radians) due to the twist applied to the muscle. The derivation of the second term in the u_1 and u_2 equations of equation (1) above corresponding to twist is given in the Appendix.

The strain E_{ij} in linear theory of elasticity is given by

$$E_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2}$$

where the comma denotes partial derivative and $i, j = 1, 2, 3$. Here, strain is defined as change in length/original length.

Thus using (1) and (2), we obtain

$$\begin{aligned} &E_{11} = k_3 - 1 + \left(Cos\psi - 1\right) \quad E_{22} = k_2 - 1 + \left(Cos\psi - 1\right) \quad E_{33} = k_4 \\ &E_{12} = E_{21} = E_{23} = E_{32} = 0, \quad E_{13} = E_{31} = \frac{k_1}{2} \end{aligned} \tag{3}$$

Henceforth, we shall designate the term $\left(Cos\psi - 1\right)$ (corresponding to twist) as k_5.

2.1 Basic Field Equations for Elastic Skeletal Muscle (Elastic Model)

The loads needed to produce the above type of deformation can be determined by evaluating the stresses in the biceps muscle. Stress is defined as load/ area of cross section on which load is applied.

The biceps muscle is taken to be a transversely isotropic material [9]. Transversely isotropic material is defined as material in which there is a specific direction (axis of transverse isotropy) perpendicular to which the elastic properties are the same but are different from that of the axis of transverse isotropy. (see Fig.2). The stress- strain relation for transversely isotropic material [10] valid for the biceps muscle tissue is given by:

$$\begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2Y_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2Y_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2Y_{12} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ E_{23} \\ E_{31} \\ E_{12} \end{bmatrix} \tag{4}$$

Here,

$$C_{11} = \left(\frac{Y_1}{1+\nu_{21}}\right)\frac{\left(1-\nu_{31}{}^2\left(Y_1/Y_3\right)\right)}{D}, \quad C_{12} = \left(\frac{Y_1}{1+\nu_{21}}\right)\frac{\left(\nu_{21}+\nu_{31}{}^2\left(Y_1/Y_3\right)\right)}{D}$$

$$C_{13} = \frac{Y_1\nu_{31}}{D}, \quad C_{33} = Y_3 + 2\nu_{31}C_{13}, \quad D = 1-\nu_{21}-2\nu_{31}{}^2\left(\frac{Y_1}{Y_3}\right) > 0$$

We take for biological tissues[9,11]

$\nu_{31} = \nu_{32} = 0.45,$ for D >0, and $\nu_{12} = \left(1-\frac{\nu_{31}Y_1}{Y_3}\right)$. Then $C_{33} = C_{13}$.

where C_{ij} are the elastic coefficients and ν_{ij} are Poisson's ratios. T_{ij} and E_{ij} are stress and strain components. Y_1 and Y_3 are Young's moduli in the X_1 and X_3 directions,

respectively, and Y_{13}, Y_{12} are the shear moduli. ν_{31} is the Poisson's ratio for the transverse strain in the X_1 or X_2 direction when stressed in the X_3 direction. Similarly, $\nu_{12} = \nu_{21}$ is the Poisson's ratio in the plane of isotropy.

Thus from (3), and (4), the stresses T_{ij} are given by

$$\begin{aligned}
T_{11} &= C_{11}E_{11} + C_{12}E_{22} + C_{13}E_{33} \\
T_{22} &= C_{12}E_{11} + C_{11}E_{22} + C_{13}E_{33} \\
T_{33} &= C_{13}E_{11} + C_{13}E_{22} + C_{13}E_{33} \\
T_{23} &= T_{32} = 2Y_{13}E_{23} = 0 \\
T_{31} &= T_{13} = 2Y_{13}E_{13} \\
T_{12} &= T_{21} = 2Y_{12}E_{12} = 0
\end{aligned} \tag{5}$$

Since all these stresses are constants, equations of equilibrium are automatically satisfied. Now we consider the following cases.

(i) Compression and shear only
(ii) Extension only
(iii) Compression and twist only

For case (i), i.e., Compression and shear along the negative X_1 axis, we must have $T_{22} = T_{33} = 0$.

Then, from (5), we get $E_{22} = \dfrac{C_{13} - C_{12}}{C_{11} - C_{13}} E_{11}$, $E_{33} = \dfrac{C_{12} - C_{11}}{C_{11} - C_{13}}$ (6)

This makes $T_{11} = \left[C_{11} + C_{12}\left(\dfrac{C_{13} - C_{12}}{C_{11} - C_{13}}\right) + C_{13}\left(\dfrac{C_{12} - C_{11}}{C_{11} - C_{13}}\right)\right] E_{11}$ (7)

and $T_{13} = 2Y_{13}E_{13}$ (8)

For case (ii), i.e., for extension along the X_3 axis, we must have $T_{22} = T_{11} = 0$. Then, from (5), we get

$$E_{22} = E_{11} = \frac{-C_{13}E_{33}}{C_{11} + C_{12}}. \tag{9}$$

This makes $T_{33} = C_{13}\left[1 - \frac{2C_{13}}{C_{11} + C_{12}}\right]E_{33}$. (10)

For case (iii), i.e., for twist about the X_3 axis and compression on the faces

$X_1 = a,\ \ X_2 = b,$ we must have $T_{33} = 0.$ Then, from (5) and (3), we have

$$E_{33} = -\left(E_{11} + E_{22}\right),\ E_{11} = E_{22}, k_1 = 0 \tag{11}$$

$$\begin{aligned} T_{11} &= \left(C_{11} - C_{13}\right)E_{11} + \left(C_{12} - C_{13}\right)E_{22} \\ T_{22} &= \left(C_{12} - C_{13}\right)E_{11} + \left(C_{11} - C_{13}\right)E_{22} \\ T_{33} &= T_{13} = T_{23} = 0 \end{aligned} \tag{12}$$

The twisting moment *M*, applied to the muscle is given by the equation:

$$M = \iint T_{11}.X_2\, dX_2\, dX_3 + \iint T_{22}\, dX_1\, dX_3$$

The limits of X_1 are from 0 to a, of X_2 from $-b$ to $+b$ and of X_3 from 0 to c

Thus, *M* per unit area becomes $\frac{T_{22}a}{2}$ where *a* is the thickness of the muscle, i.e.,

$$M = \frac{T_{22}a}{2} \tag{13}$$

2.2 Basic Field Equations for Viscoelastic Skeletal Muscle (Viscoelastic Model)

In order to determine the dynamic stress responding to the applied dynamic extension for viscoelastic fasciae, a quasi-linear constitutive equation [12] is employed to address the viscoelastic behavior in biological soft tissues. This can be written as:

$$T_{ij}(\mathrm{x},t) = \int_{-\infty}^{t} G_{ijkl}(t-\tau)\frac{\partial T_{kl}^{(e)}(\mathrm{x},\tau)}{\partial \tau}d\tau \tag{14}$$

where T^e is the pseudo-elastic stress and a function of strain, which in turn is a function of position x and time *t*; G_{ijkl} (i,j,k,l =1,2,3---6) is the tensor of relaxation function; τ is a dummy variable for the integration from the beginning (t=0) of the deformation to the present time t. At this stage we designate

$T_1, T_2, ..., T_6$ for $T_{11}, T_{22}, T_{33,} T_{23}, T_{31}, T_{12}$ and $E_1, E_2, ..., E_6$ for $E_{11,} E_{22}, E_{33}, E_{23}, E_{31}, E_{12.}$

Then (14) can be rewritten as

$$T_n = \int_0^t G_{nq}(t-\tau)\frac{\partial T^e}{\partial \tau}\, d\tau \quad (n,q = 1,2,...,6) \tag{15}$$

In (15), the uninvolved diagonal relaxation functions are assumed to be zero and we retain $G_{11}, G_{22}, G_{33}, G_{13}$ .Then we get

$$T_1 = \int_0^t \left(G_{11}(t-\tau)\frac{\partial T_1}{\partial \tau} + G_{13}(t-\tau)\frac{\partial T_3}{\partial \tau} \right) d\tau$$

$$T_2 = \int_0^t \left(G_{22}(t-\tau)\frac{\partial T_2}{\partial \tau} \right) d\tau \tag{16}$$

$$T_3 = \int_0^t \left(G_{31}(t-\tau)\frac{\partial T_1}{\partial \tau} + G_{33}(t-\tau)\frac{\partial T_3}{\partial \tau} \right) d\tau$$

The final step is to determine the relaxation function. A one-dimensional standard model was introduced in [13] to derive a linear reduced relaxation function with a continuous spectrum. The widely accepted reduced relaxation function [13] is given as:

$$G(t) = \frac{1 + \int_0^\infty S(q)\, e^{-t/q} dq}{1 + \int_0^\infty S(q)\, dq}\,. \tag{17}$$

The relaxation spectrum $S(q)$ denotes the amplitude of the viscous dissipation subject to the frequency $1/q$. For many biological materials that exhibit viscoelastic behavior and low sensitivity to strain rates, the designation of the spectrum with constant amplitude over a range of frequency is found to fit the experimental results fairly well [13]. The spectrum assumes the following form:

$$\begin{aligned} S(q) &= \frac{C0}{q}, \quad \text{for } q_{1C} \le q \le q_{2C} \\ S(q) &= 0, \quad \text{otherwise} \end{aligned} \tag{18}$$

The parameters C0, q_{1C} and q_{2C} used in Eq. (18) can be determined experimentally by examining the stress relaxation of the tissues under constant strain or by experiments of dynamic loading. In this study, we adopt the parameters obtained from the study of subcutaneous tissues of rats [14]. It was reported in [14], that C0=0.25, q_{1C}=1.86 second, and q_{2C}=110.4 second. Combining Eq. (15) – Eq.(18), one can solve numerically for the viscoelastic stress-time, stress-strain relations for the biceps muscle.

Since the viscosity is uniform in the muscle tissues, we assume $G_{11}(t-\tau) = G_{22}(t-\tau) = G_{33}(t-\tau) = G_{13}(t-\tau) = G(t-\tau)$, in (18), and take the value of the parameters, C0=0.25, q_{1C}=1.86 seconds, and q_{2C}=110.4 seconds in $G(t-\tau)$ as in [14].

3. Results

Since the elastic parameters such as the Young's moduli and the shear modulus for transversely isotropic biceps has already been determined in [9], we have used these to evaluate the mechanical stresses in manual therapy on biceps muscle. The values of these parameters are given below.

$$Y_1 = 13 \pm 4.5\text{kPa}, \quad Y_3 = 185 \pm 60\text{kPa}, \; \text{Y}_{13} = 54 \pm 3\text{kPa} \tag{19}$$

where Y_1 and Y_3 are Young's moduli along the X_1 and X_3 directions, respectively, i.e. perpendicular and parallel to muscle fibers respectively. Y_{13} is the shear modulus parallel to muscle fibers.

The following Linear and Parabolic regime simulations for strain E are used to evaluate the stresses.:

Linear regime:

$E = \frac{At}{T_0}$, i.e., strain increases linearly with time t. (20)

Parabolic regime:

$E = A\left[1 - \left(\frac{t - T_0}{T_0}\right)^2\right]$, i.e., strain varies parabolically with time t. (21)

Here, T_0 is the time for which manual therapy is performed in one stroke and A is the maximum strain, which is attained at that time.

Note that in order to evaluate the stresses in (16) for the viscoelastic model, we use the values of stresses in parentheses of the integrals in (16), as obtained from the Elastic Model.

We provide the results (Table 1) in terms of stresses, since the area of cross section and the thickness of the muscle varies with the subject, for both the elastic and viscoelastic models.

Table 1

Applied Compression 10% and Applied Shear 10% (on Face X_1=a) Applied Extension parallel to fiber direction (on Face X_3=c)		Elastic Model	Viscoelastic Model (Parabolic Model)
1. Compression and Shear only $k_3=0.9;\ k_1=0.1;\ k_5=0$	Compressive stress (N) Shear stress (T)	N = -1.30 kPa = -.189 lbs/in^2 T = 10.8 kPa = 1.566 lbs/in^2	N = -.799 kPa = -.116 lbs/in^2 T = 6.64 kPa = .963 lbs/in^2
2. Longitudinal extension only $k_4=0.1;\ k_1=0;\ k_5=0;\ k_3=1$	Longitudinal stress (F)	F = 18.5 kPa = 2.68 lbs/in^2	F = 11.4 kPa = 1.65 lbs/in^2
3. Twist and compression only $\psi = 10^o;\ k_2= k_3=0.9;\ k_1=0$	Twisting Moment per unit area about the X_3 axis (M)	M = 23.7 kPa-cm	M =14.6 kPa -cm

Note: For case 1, k_2 and k_4 can be determined from eq. (6). For case 2, k_2 can be determined from eq. (9). For case 3, k_4 can be determined from eq. (11).

The stress relaxation plots for compression and shear combined, by maintaining 10% strain, are given in Fig.3, while that for extension only are given in Fig. 4. The plot of stress relaxation, for moment with twist and compression combined, is given in Fig. 5. From Fig.3, we observe that for the curve with the cusp, compression and shear grow linearly until they reach the level of 10% at time of 60 seconds. Compression and shear are then held at the 10% level for the remainder of the 200 second simulation. We note that the stress reaches its maximum at the same time that compression and shear reach their maximum for this case. The smooth curve represents the case of parabolic transition from no compression and no shear to 10% compression and shear over 60 seconds and then the strain being held constant at that level for the remainder of the simulation. We note that in this case, the maximum stress is slightly smaller (7%) than for the linear

extension regime and that the maximum stress occurs before the full compression and shear occurs. Stress relaxes by 18% of its maximum value during the period where compression is held constant. This agrees with the concept that stress should not reduce to zero for a viscoelastic solid.

In Fig.4, four cases are presented. The solid curves correspond to the cases presented in the previous figure in that the strain in the muscle was increased from zero to 10% over 60 seconds in a linear (thick curve) and in a parabolic (thin curve) manner, respectively. In addition, the extension was performed over a period of 30 seconds (dashed curves) before being held at the level of 10% extension for the remainder of the 200 second simulation. Again, stresses reach their maxima at the same time that extension reaches its maximum in the linear extension regime. Again, the maximum stresses are slightly smaller in the parabolic extension regime than for the linear extension regime and these maximum stresses occur before the full extension occurs. As in Fig. 3, stress relaxes by 18% of its maximum value during the period where extension is held constant. The figure also demonstrates that as the run-up time to full extension shortens, the maximum stress increases.

We have performed more extensive simulations for shorter times to full extension in the linear regime. We hypothesize that the maximum stress should approach the stress we found earlier for the linear elastic analysis (without considering viscosity) as the time to full extension is decreased. That is, if extension were to take place instantaneously, there would be no time over which viscosity could act. We found indeed that this was the case, with stress (T_{33}) rising to approximately 18.2 kPa (the elastic value being 18.5kPa) for time of 0.25 seconds to full extension. See Table 2 also.

Table 2

Time (seconds)	Viscoelastic stress produced (kPa)
60	12.0
30	13.2
5	16.5
2	17.0
0.25	18.2

In Fig.5, twisting moment (M) relaxation is presented vs. time for the case of simultaneous compression (up to 10% strain) and twist (up to 10 degrees). The curves correspond to the cases where compression and twist were increased from zero to their maximum values over 60 seconds in a linear (thick curve) and in a parabolic (thin curve) manner, respectively, and then the strain and twist angle were held constant for the remainder of the 200 second simulations. Just as in the compression and shear and in the pure extension cases, the moment reaches its maximum at the same time that compression and twist angle reach their maximum values in the linear extension regime. Also, the moment relaxes by 18% of its maximum value during the time compression and twist angle are held constant. Again, the maximum moment is slightly smaller in the parabolic extension regime than for the linear extension regime and the maximum moment occurs before the full compression and twist occur.

Discussion

The linear theory of elasticity has been used for investigating the loads in manual therapy on biceps muscle, although the muscle tissue exhibits non-linear behavior. Since the micro-failure region in which the tissue release is felt in manual therapy occurs in the linear region [15], the linear theory is applicable.

This analysis is valid for in vivo state and transversely isotropic biceps muscle, as we have used the mechanical constants determined experimentally for in vivo state and used the stress-strain relation for transversely isotropic material.

The results are very sensitive to the value of the Poisson's ratio ν_{31}.

We have used a parabolic trajectory for the value of the strain, E, to go from 0 to A in time T0. This corresponds to the value of N =1 in the form of the equation for E, given by:

$$E=A\left(1-\left(\frac{t-T0}{T0}\right)^{2N}\right)$$

We can use N = 2, 3 which will correspond to a quartic, sixth degree trajectory and so forth.

Conclusions.

The applied loads for producing compression, shear, extension and twist in manual therapy on biceps muscle are determined from mathematical analysis. This analysis is valid for in vivo state and transversely isotropic and viscoelastic biceps muscle. Since, linear theory of elasticity is used in this analysis, the results for any percentage deformation can be obtained by linear interpolation from the results of 10% deformation in this analysis which are given in table1.

The results indicate that with quick maneuvers, the viscoelasticity effect is decreased, i.e. 50% greater load needs to be applied .

80% of the maximal stress is produced during the first 46 seconds of 60 seconds duration of manual therapy

The maximum stress attained for the same amount of deformation, (i.e. 10% in this analysis) using the viscoelastic model is 7% less compared to the elastic model. In other words, less load is needed to produce the same amount of deformation if the biceps muscle is considered as viscoelastic tissue compared to elastic tissue.

In our previous paper [16] dealing with the fasciae, we found that due to the stiffness of the fasciae, no deformation was produced under physiological loads. However, something must yield beneath the fasciae. From the present analysis on biceps muscle, we conclude that it is the muscle underneath which yields, since very small loads are needed to deform the muscle.

Since Young's modulus parallel to muscle fibers is about 15 times as high as that along the perpendicular direction, the biceps muscle is 15 times stiffer in the directions parallel to the muscle fibers compared to the perpendicular direction.

We hope that the mathematical analysis performed in this paper will strengthen the scientific foundation for understanding of osteopathic manual treatment.

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Figure Captions

Fig. 1 Skeletal Muscle Element.

OX_1, OX_2, OX_3 are the axes in the Cartesian coordinates system, M denotes the moment, N and T denote the normal pressure and tangential force, respectively.

Fig.2. Transversely Isotropic Biceps Muscle.

x_1, x_2 plane is a plane of isotropy, i.e. in this plane elastic moduli are same in all directions. Elastic property along x_3 axis is different from the elastic properties in x_1, x_2 plane.

Fig.3. Stress relaxation for T_{11} i.e. normal pressure N, vs. time for compression and shear of 10% for linear and parabolic strain trajectories. The thick solid curve corresponds to a linear trajectory while the thin solid curve corresponds to a parabolic trajectory.

Fig.4. Stress relaxation for T_{33} i.e. F, vs. time for pure extension of 10% for linear and parabolic strain trajectories. The thick solid and dashed curves correspond to slow and fast linear trajectories, respectively. The thin solid and dashed curves correspond to slow and fast parabolic trajectories, respectively.

Fig.5. Stress relaxation for twisting moment (M) vs. time for the case of simultaneous compression (up to 10%) on the faces $X_1 = a$; $X_2 = b$ and twist (up to 10 degrees) about the X_3 axis. The thick solid curve corresponds to a linear trajectory while the thin solid curve corresponds to a parabolic trajectory.

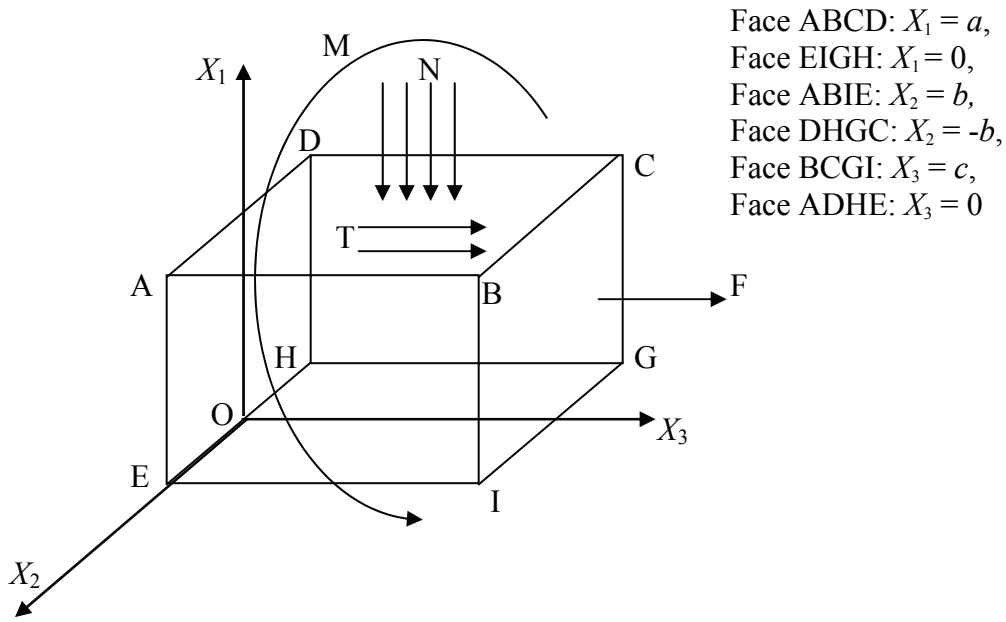

Fig.1 Skeletal Muscle Element

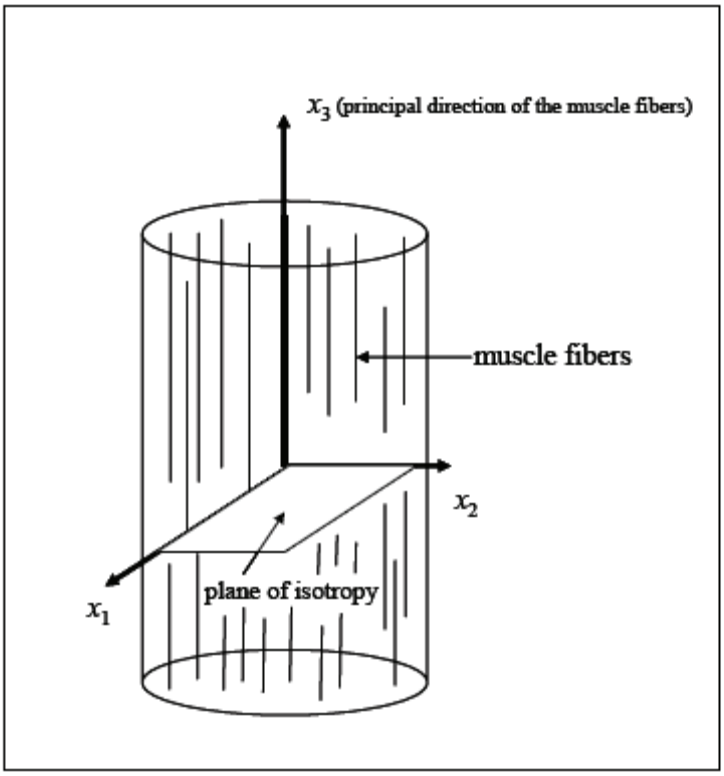

Fig. 2. Transversely Isotropic Biceps Muscle

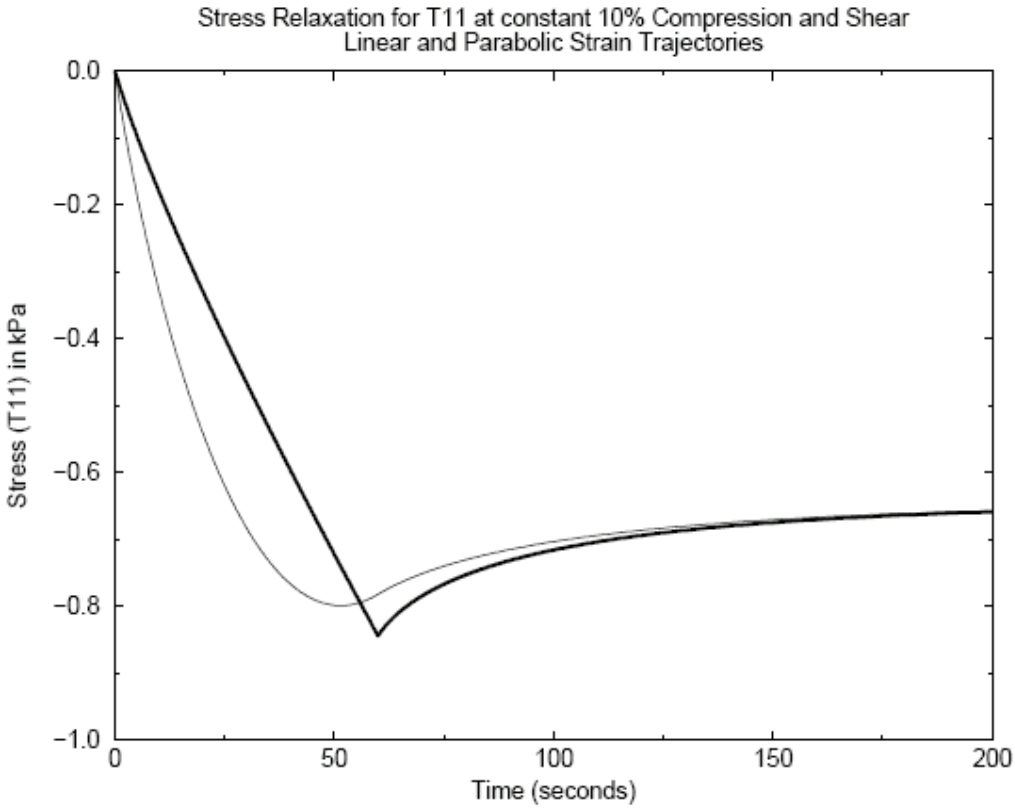

Fig. 3. Stress relaxation for T_{11}, i.e. normal pressure N, vs. time for compression and shear of 10% for linear and parabolic strain trajectories

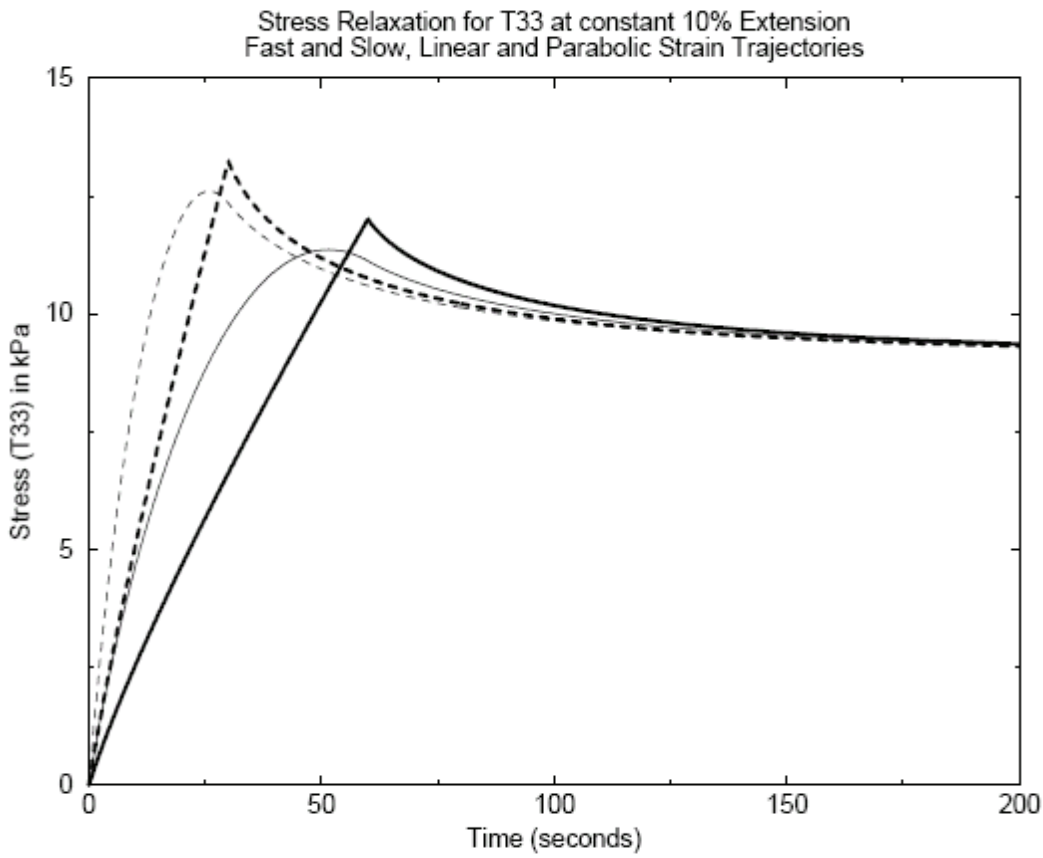

Fig.4. Stress relaxation for T_{33}, i.e. F, vs. time for pure extension of 10% for linear and parabolic strain trajectories

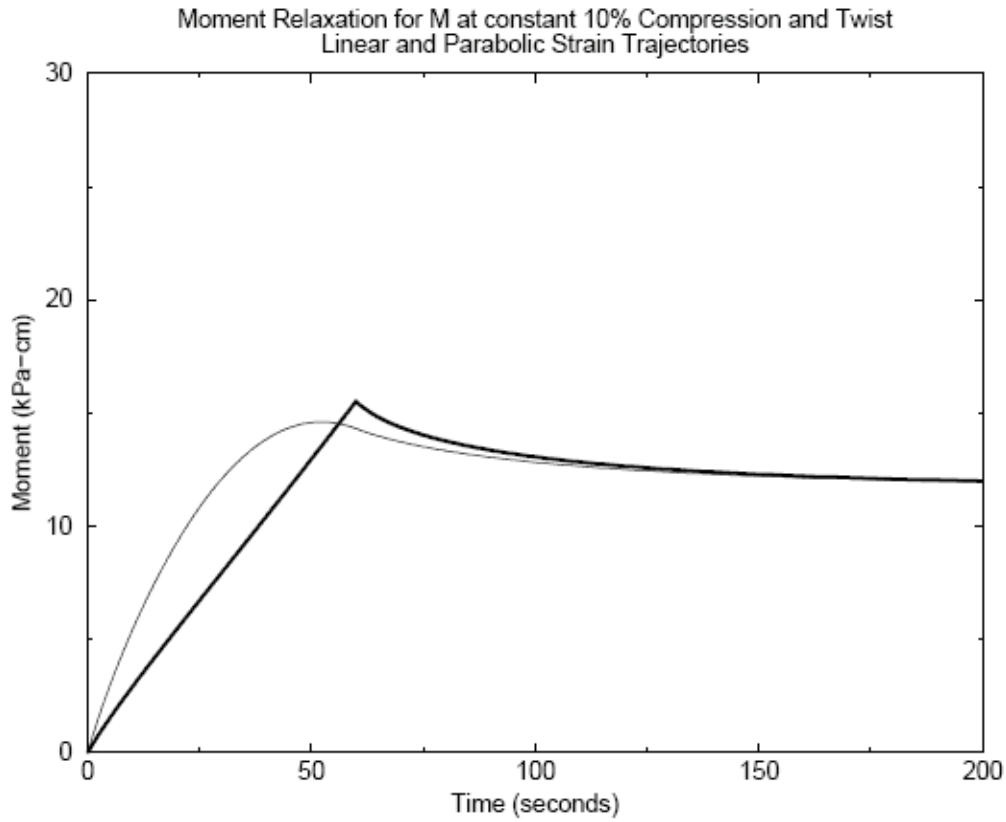

Fig.5 Stress relaxation for twisting moment (M) vs. time for the case of simultaneous compression (up to 10%) on the faces $X_1 = a$; $X_2 = b$ and twist (up to 10 degrees) about the X_3 axis

Appendix

Derivation of second term in the u_1 and u_2 equations of equation (1) of the text

Let θ be the initial angle of any point (r,θ) in X_1, X_2 plane, and ψ be the angle of twist. Then the displacements u_1, u_2 are given by

$$u_1 = r\,Cos(\theta+\psi) - r\,Cos\,\theta$$
$$u_2 = r\,Sin\,(\theta+\psi) - r\,Sin\,\theta$$

Expressed in terms of X_1, X_2, these become

$X_1 Cos\psi - X_2 Sin\psi - X_1$ and $X_2 Cos\psi + X_1 Sin\psi - X_2$. These are the second terms in u_1 and u_2 equations of equation (1) in the text.